

Table of Laplace Transforms

| $f(t) = \mathcal{L}^{-1}\{F(s)\}$ | $F(s) = \mathcal{L}\{f(t)\}$ | $f(t) = \mathcal{L}^{-1}\{F(s)\}$ | $F(s) = \mathcal{L}\{f(t)\}$ |
|---|--|---|---|
| 1. 1 | $\frac{1}{s}$ | 2. e^{at} | $\frac{1}{s-a}$ |
| 3. $t^n, n=1,2,3,\dots$ | $\frac{n!}{s^{n+1}}$ | 4. $t^p, p > -1$ | $\frac{\Gamma(p+1)}{s^{p+1}}$ |
| 5. \sqrt{t} | $\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$ | 6. $t^{n-\frac{1}{2}}, n=1,2,3,\dots$ | $\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$ |
| 7. $\sin(at)$ | $\frac{a}{s^2+a^2}$ | 8. $\cos(at)$ | $\frac{s}{s^2+a^2}$ |
| 9. $t \sin(at)$ | $\frac{2as}{(s^2+a^2)^2}$ | 10. $t \cos(at)$ | $\frac{s^2-a^2}{(s^2+a^2)^2}$ |
| 11. $\sin(at) - at \cos(at)$ | $\frac{2a^3}{(s^2+a^2)^2}$ | 12. $\sin(at) + at \cos(at)$ | $\frac{2as^2}{(s^2+a^2)^2}$ |
| 13. $\cos(at) - at \sin(at)$ | $\frac{s(s^2-a^2)}{(s^2+a^2)^2}$ | 14. $\cos(at) + at \sin(at)$ | $\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$ |
| 15. $\sin(at+b)$ | $\frac{s \sin(b) + a \cos(b)}{s^2+a^2}$ | 16. $\cos(at+b)$ | $\frac{s \cos(b) - a \sin(b)}{s^2+a^2}$ |
| 17. $\sinh(at)$ | $\frac{a}{s^2-a^2}$ | 18. $\cosh(at)$ | $\frac{s}{s^2-a^2}$ |
| 19. $e^{at} \sin(bt)$ | $\frac{b}{(s-a)^2+b^2}$ | 20. $e^{at} \cos(bt)$ | $\frac{s-a}{(s-a)^2+b^2}$ |
| 21. $e^{at} \sinh(bt)$ | $\frac{b}{(s-a)^2-b^2}$ | 22. $e^{at} \cosh(bt)$ | $\frac{s-a}{(s-a)^2-b^2}$ |
| 23. $t^n e^{at}, n=1,2,3,\dots$ | $\frac{n!}{(s-a)^{n+1}}$ | 24. $f(ct)$ | $\frac{1}{c} F\left(\frac{s}{c}\right)$ |
| 25. $u_c(t) = u(t-c)$ Heaviside Function | $\frac{e^{-cs}}{s}$ | 26. $\delta(t-c)$ Dirac Delta Function | e^{-cs} |
| 27. $u_c(t) f(t-c)$ | $e^{-cs} F(s)$ | 28. $u_c(t) g(t)$ | $e^{-cs} \mathcal{L}\{g(t+c)\}$ |
| 29. $e^{ct} f(t)$ | $F(s-c)$ | 30. $t^n f(t), n=1,2,3,\dots$ | $(-1)^n F^{(n)}(s)$ |
| 31. $\frac{1}{t} f(t)$ | $\int_s^\infty F(u) du$ | 32. $\int_0^t f(v) dv$ | $\frac{F(s)}{s}$ |
| 33. $\int_0^t f(t-\tau) g(\tau) d\tau$ | $F(s)G(s)$ | 34. $f(t+T) = f(t)$ | $\frac{\int_0^T e^{-st} f(t) dt}{1-e^{-sT}}$ |
| 35. $f'(t)$ | $sF(s) - f(0)$ | 36. $f''(t)$ | $s^2 F(s) - sf(0) - f'(0)$ |
| 37. $f^{(n)}(t)$ | $s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0)$ | | |

Table Notes

1. This list is not inclusive and only contains some of the more commonly used Laplace transforms and formulas.
2. Recall the definition of hyperbolic trig functions.

$$\cosh(t) = \frac{e^t + e^{-t}}{2} \qquad \sinh(t) = \frac{e^t - e^{-t}}{2}$$

3. Be careful when using “normal” trig function vs. hyperbolic trig functions. The only difference in the formulas is the “+ a²” for the “normal” trig functions becomes a “- a²” for the hyperbolic trig functions!
4. Formula #4 uses the Gamma function which is defined as

$$\Gamma(t) = \int_0^{\infty} e^{-x} x^{t-1} dx$$

If n is a positive integer then,

$$\Gamma(n+1) = n!$$

The Gamma function is an extension of the normal factorial function. Here are a couple of quick facts for the Gamma function

$$\Gamma(p+1) = p\Gamma(p)$$

$$p(p+1)(p+2)\cdots(p+n-1) = \frac{\Gamma(p+n)}{\Gamma(p)}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$