Series Cheatsheet

Definitions

Basic Series

Infinite Sequence: $\langle s_n \rangle$

Limit/Convergence of a Sequence: $\lim_{n\to\infty} s_n = L$

Infinite Serie: (Partial sums) $S_n = \sum s_n = s_1 + s_2 + \cdots + s_n + \cdots$

Geometric Serie:

$$\sum_{k=1}^{n} ar^{k-1} = S_n = a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

Positive Series

Positive Serie: If all the terms s_n are positive.

Integral Test: If $f(n) = s_n$, continuous, positive, decreasing: $\sum s_n$ converges $\iff \int_1^\infty f(x)dx$ converges.

Comparison Test: $\sum a_n$ and $\sum b_n$ where $a_k < b_k \quad (\forall k \geq m)$

1. If $\sum b_n$ converges, so does $\sum a_n$
2. If $\sum a_n$ diverges, so does $\sum b_n$

Limit Comparison Test: $\sum a_n$ and $\sum b_n$ such that $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists, $\sum a_n$ converges $\iff \sum b_n$ converges.

Convergence

Alternating Serie:

$$\sum (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + a_5 - \cdots$$

Absolute Convergence: If $\sum |s_n|$ is convergent.

Conditional Convergence: If $\sum s_n$ is convergent but *not* absolutely convergent.

Ratio Test: If $\lim_{n\to\infty} |\frac{s_{n+1}}{s_n}| =$

- < 1: absolutely convergent
- 1: (no conclusion)
- > 1 or $+\infty$: diverges

Root Test: If $\lim_{n\to\infty} \sqrt[n]{|s_n|} =$

- < 1: absolutely convergent
- 1: (no conclusion)
- > 1 or $+\infty$: diverges

Uniform Convergence: If $\forall \epsilon > 0, \exists m$ such that for each x and every $n \geq m$, $\quad f_n(x) - f(x) < \epsilon$

Power Series

Power Serie:

$$\sum_{n=0}^{+\infty} a_n(x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots$$

Power Serie About Zero:

$$\sum_{n=0}^{+\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots$$

Taylor Serie
If f a function infinitely differentiable,

$$\sum_{n=0}^{+\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

MacLaurin Serie
If f a function infinitely differentiable,

$$\sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!}x^n$$

Taylor's Formula with Remainder
$\exists x^*$ between c and x such that

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k + R_n(x)$$

$$R_n(x) = \frac{f^{(n+1)}(x^*)}{(n+1)!}(x-c)^{n+1}$$

Applications

Application: Showing Function/Taylor-Series Equivalence

$$\lim_{n\to+\infty} R_n(x) = 0$$

Application: Approximating Functions or Integrals

$$R_n(x_0) < K$$

Binomial Serie

$$(1+x)^r = 1 + \sum_{n=1}^{+\infty} \frac{r(r-1)(r-2)\cdots(r-n+1)}{n!}x^n$$

Common Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots +$$

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 +$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$$