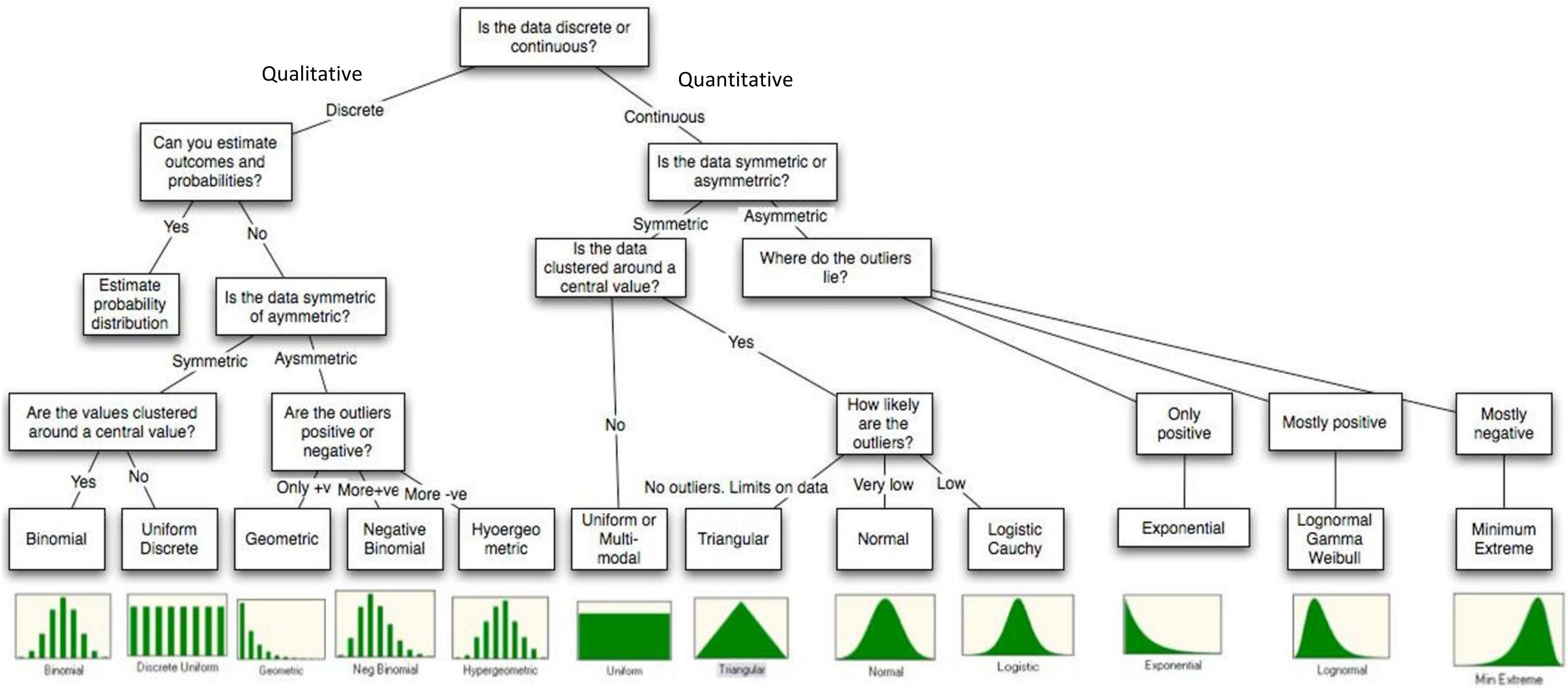
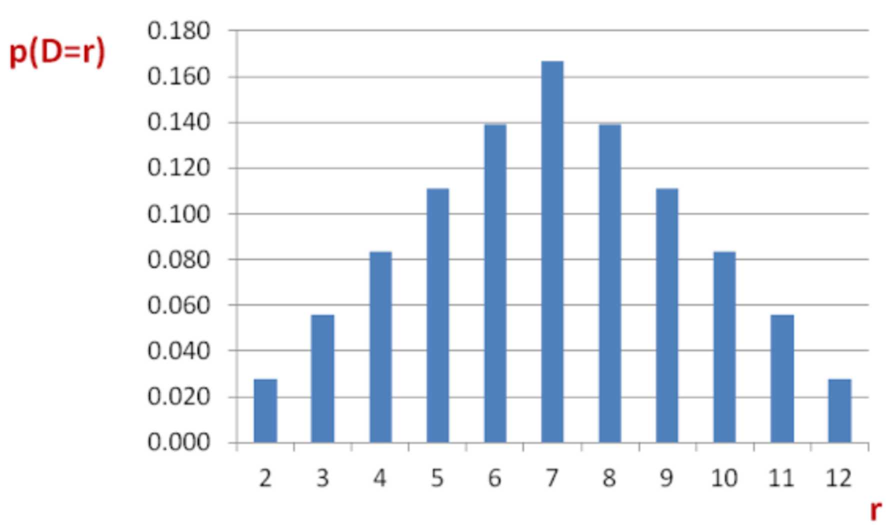
Harold's Statistical Distributions
Cheat Sheet
22 October 2022

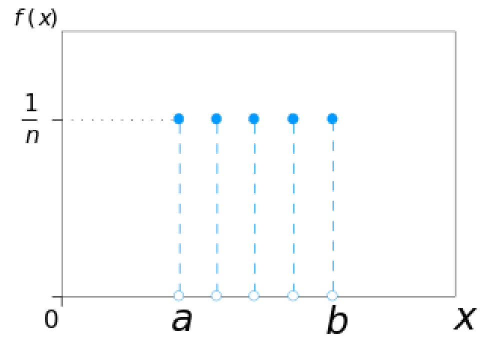
PDF Selection Tree to Describe a Single Population

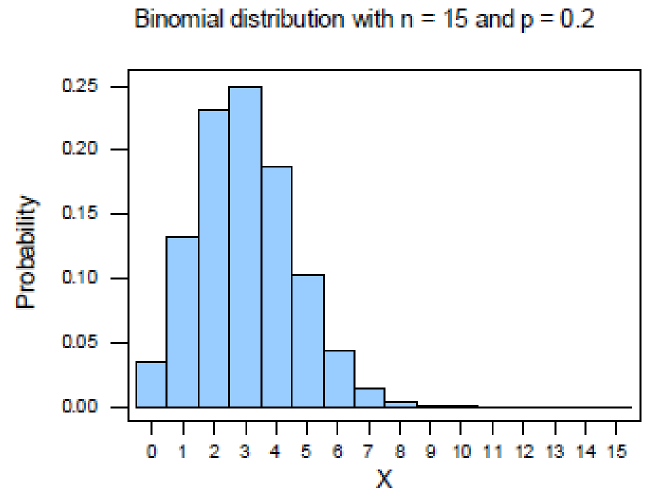


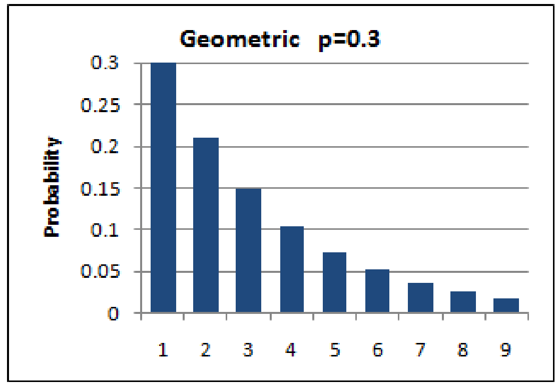
Discrete Definitions

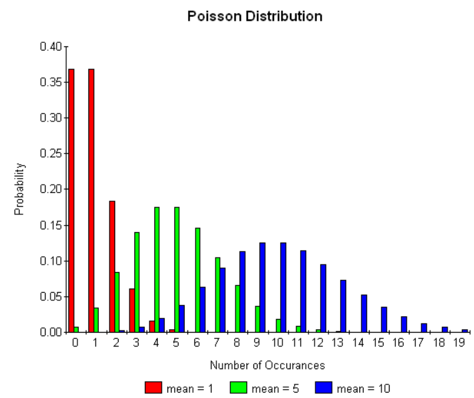
Term	Definition	Description
Random Variable	X 	A rule that assigns a number to every **outcome** in the sample space, S. $$X(a,b) = a + b = r$$ Example: Sum of a pair of dice $$X(2,4) = 2 + 4 = 6$$ Derived from a probability experiment with different probabilities for each X. **Used in discrete or finite PDFs.**
Event	$X = r$ $X(s) = r$	An event assigns a value to the random variable X with probability: $$P(X = r)$$ Example: Sum of a pair of dice $$P(X = 6) = \frac{5}{36}$$
Distribution	The distribution of a random variable is the set of all pairs *(r, p(X = r))* such that *r ∈X(S)*.	Set of all outcomes with their probabilities. $$(r, p(X = r))$$ Example: Sums of all pair of dice $$\left\{\left(2, \frac{1}{36}\right), \left(3, \frac{1}{18}\right), \ldots, \left(12, \frac{1}{36}\right)\right\}$$
Sum of Probabilities	$$\sum_{r \in X(S)} P(X = r) = 1$$	A random variable has some fractional probability value for every outcome in the sample space.
Histogram	Histogram of the distribution over the sum of two dice: 	
PMF	Probability Mass Function	Discrete, Qualitative
PDF	Probability Density Function	Continuous, Quantitative

Discrete Probability Mass Functions (Qualitative)

Probability Mass Function (PMF)	Mean	Standard Deviation
Uniform Discrete Distribution		
$P(X = x) = \frac{1}{b - a + 1}$	$\mu = \frac{a + b}{2}$	$\sigma = \sqrt{\frac{(b - a)^2}{12}}$
Conditions	• All outcomes are consecutive. • All outcomes are equally likely. • Not common in nature.	
Variables	*a* = minimum *b* = maximum	
TI-84	NA	
Example	Tossing a fair die (n = 6)	
Online PDF Calculator	http://www.danielsoper.com/statcalc3/calc.aspx?id=102	

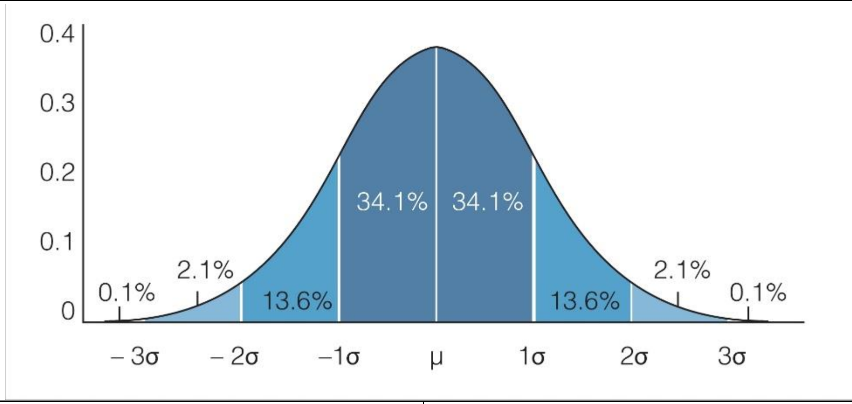
Probability Mass Function (PMF)	Mean	Standard Deviation
Binomial Distribution		
$X \sim B(n,p) = B(k;n,p) =$ $\boldsymbol{P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}}$ where $\binom{n}{k} = {}_nC_k = \frac{n!}{k!\,(n-k)!}$	$\boldsymbol{\mu_x = np}$ $\boldsymbol{\mu_{\hat{p}} = p}$ *p* = probability of success *q = (1 – p)* = probability of failure	$\boldsymbol{\sigma_x = \sqrt{np(1-p)}} = \sqrt{npq}$ $np \geq 10$ **and** $nq \geq 10$ $\boldsymbol{\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}} = \sqrt{\frac{pq}{n}}$
$P(X=k) \approx \frac{1}{\sqrt{npq}} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\frac{(k-np)^2}{npq}}$	Use for large n (> 15) to approximate binomial distribution.	
Conditions	• n is fixed. • The probabilities of success (p) and failure (q) are constant. • Each trial is independent.	
Variables	*n* = fixed number of trials *p* = probability that the designated event occurs on a given trial (Symmetric if *p = 0.5*) X = Total number of times the event occurs $(0 \leq X \leq n)$	
TI-84	For one x value: [2nd] [DISTR] A:binompdf(n,p,x) $P(X=k)$ [2nd] [DISTR] B:binomcdf(n,p,x) $P(X \leq k)$ For a range of x values [j,k]: [2nd] [DISTR] A:binompdf([ENTER] n, p, [↓] [↓] [ENTER] [STO>] [2nd] [3] (=L3) [ENTER] [2nd] [LIST] [→→ MATH] 5:sum(L3,j+1,k+1)	
Example	*Larry's batting average is 0.260. If he's at bat four times, what is the probability that he gets exactly two hits?* Solution: n = 4, p = 0.26, x = 2 binompdf(4,0.26,2) = 0.2221 = 22.2%	
Online PDF Calculator	http://stattrek.com/online-calculator/binomial.aspx	

Probability Mass Function (PMF)	Mean	Standard Deviation
Geometric Distribution		
$P(X \leq x) = q^{x-1}p = (1-p)^{x-1}p$ $P(X > x) = q^x = (1-p)^x$	$\mu = E(X) = \frac{\mathbf{1}}{\boldsymbol{p}}$	$\boldsymbol{\sigma} = \frac{\sqrt{q}}{p} = \sqrt{\frac{\mathbf{1} - \boldsymbol{p}}{\boldsymbol{p}^2}}$
Conditions	• A series of independent trials with the same probability of a given event. • Probability that it takes a specific amount of trials to get a success. Can answer two questions: a) Probability of getting 1st success on the n^{th} trial b) Probability of getting success on $\leq n$ trials Since we only count trials until the event occurs the first time, there is no need to count the ${}_nC_x$ arrangements, as in the binomial distribution.	
Variables	*p* = probability that the event occurs on a given trial *X* = # of trials until the event occurs the 1st time	
TI-84	[2nd] [DISTR] E:geometpdf(p, x) $P(X = x)$ [2nd] [DISTR] F:geometcdf(p, x) $P(X \leq x)$	
Example	*Suppose that a car with a bad starter can be started 90% of the time by turning on the ignition. What is the probability that it will take three tries to get the car started?* Solution: *p* = 0.90, *X* = 3 geometpdf(0.9, 3) = 0.009 = 0.9%	
Online PDF Calculator	http://www.calcul.com/show/calculator/geometric-distribution	

Probability Mass Function (PMF)	Mean	Standard Deviation
Poisson Distribution		
$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0,1,2,3,4,\ldots$	$\mu = E(X) = \lambda$	$\sigma = \sqrt{\lambda}$
Conditions	• Events occur independently, at some average rate per interval of time/space.	
Variables	λ = average rate X = total number of times the event occurs There is no upper limit on X	
TI-84	[2nd] [DISTR] C:poissonpdf(λ, X) $P(X = x)$ [2nd] [DISTR] D:poissoncdf(λ, X) $P(X \leq x)$	
Example	*Suppose that a household receives, on the average, 9.5 telemarketing calls per week. We want to find the probability that the household receives 6 calls this week.* Solution: λ = 9.5, X = 6 poissonpdf(9.5, 6) = 0.0764 = 7.64%	
Online PDF Calculator	http://stattrek.com/online-calculator/poisson.aspx	

Bernoulli	See http://www4.ncsu.edu/~swu6/documents/A-probability-and-statistics-cheatsheet.pdf
tnomial	
Hypergeometric	
Negative Binomial	

Continuous Probability Density Functions (Quantitative)

<table>
<tr><th>Probability Density Function (PDF)</th><th>Mean</th><th>Standard Deviation</th></tr>
<tr><td>Normal Distribution / Gaussian Distribution / Bell-Shaped Curve</td><td colspan="2">
</td></tr>
<tr><td>$X \sim \mathcal{N}(\mu, \sigma^2) = \mathcal{N}(x; \mu, \sigma^2) =$
$\Phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$</td><td>$\mu = E(x) = \mu$</td><td>$\sigma = \sigma$</td></tr>
<tr><td>Special Case: Standard Normal
$Z \sim \mathcal{N}(0, 1) = \mathcal{N}(x; 0, 1)$</td><td>$\mu = 0$</td><td>$\sigma = 1$</td></tr>
<tr><td>z-Score of a Sample</td><td colspan="2">$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$</td></tr>
<tr><td>Conditions</td><td colspan="2">• Symmetric, unbounded, bell-shaped.
• No data is perfectly normal. Instead, a distribution is approximately normal.</td></tr>
<tr><td>Variables</td><td colspan="2">μ = mean (= median = mode)
σ = standard deviation
x = observed value (all real numbers)</td></tr>
<tr><td>TI-84</td><td colspan="2">Have scores, need area: $f(x) = P(X = x)$
z-scores: [2nd] [DISTR] 1:normalpdf(z, 0, 1)
x-scores: [2nd] [DISTR] 1:normalpdf(x, μ, σ)

Have boundaries, need area: $F(x) = P(X \le x)$
z-scores: [2nd] [DISTR] 2:normalcdf(left-bound, right-bound)
x-scores: [2nd] [DISTR] 2:normalcdf(left-bound, right-bound, μ, σ)

Have area, need boundary:
z-scores: [2nd] [DISTR] 3:invNorm(area to left)
x-scores: [2nd] [DISTR] 3:invNorm(area to left, μ, σ)</td></tr>
</table>

<table>
<tr><td>Python</td><td><pre>
import scipy.stats as st

mean, sd, z = 0, 1, 1.5
print(st.norm.cdf(z, mean, sd))   # P(z <= 1.5)
print(st.norm.sf(z, mean, sd))   # P(z >= 1.5)

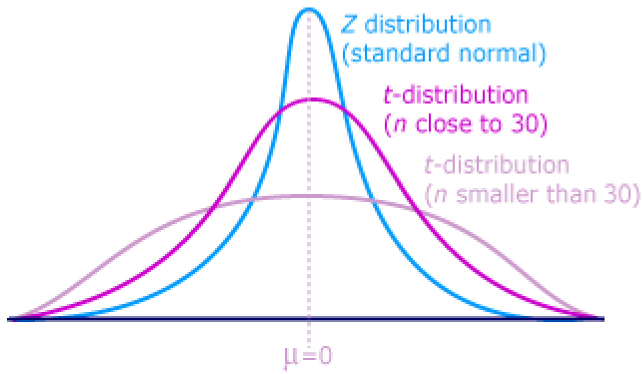
mean, sd, x = 55, 7.5, 62
print(st.norm.cdf(x, mean, sd))   # P(x <= 62)
print(st.norm.sf(x, mean, sd))   # P(x >= 62)

                                     # P(49 < t < 60)
print(st.norm.cdf(60, mean, sd) - st.norm.cdf(49,
mean, sd))
</pre></td></tr>
<tr><td>Example</td><td>Suppose the mean score on the math SAT is 500 and the standard deviation is 100. What proportion of test takers earn a score between 650 and 700?

<u>Solution:</u>
left-boundary = 650, right boundary =700, μ = 500, σ = 100
normalcdf(650, 700, 500, 100) = 0.0441 = ~4.4%</td></tr>
<tr><td>Online PDF Calculator</td><td>http://davidmlane.com/normal.html</td></tr>
</table>

Standard Normal Distribution Table: Positive Values (Right Tail) Only

Z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53980	0.54380	0.54776	0.55172	0.55567	0.55966	0.56360	0.56749	0.57142	0.57535
0.2	0.57930	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998
4.1	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999
4.2	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	1.00000

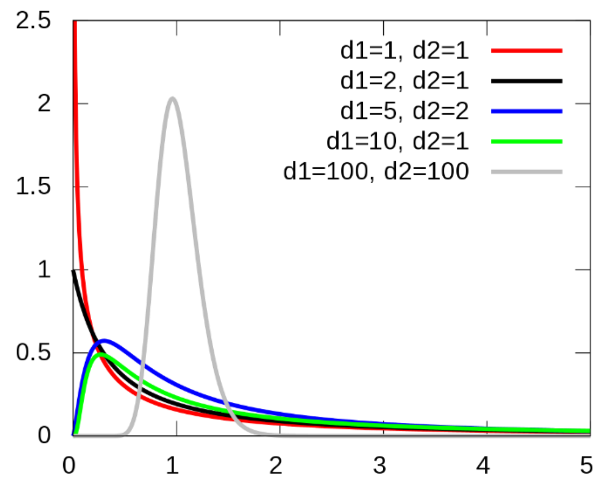
Probability Density Function (PDF)	Mean	Standard Deviation
Student's t Distribution	 This distribution was first studied by William Gosset, who published under the pseudonym *Student*. It has a wider spread because of a slightly larger standard deviation.	
Degrees of Freedom	*df* = degrees of freedom = $n-1$ A positive whole number that indicates the lack of restrictions in our calculations. The number of values in a calculation can vary. e.g., $df = 1$ means 1 equation 2 unknowns $\lim_{v\to\infty} tpdf(x, df) = normalpdf(x)$	
$P(v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{x^2}{v}\right)^{\frac{-(v+1)}{2}}$	$\mu = E(x) = 0$ (always)	$\sigma = 0$
Where the Gamma function $\Gamma(s) = \int_0^{\infty} t^{s-1}e^{-t}dt$	$\Gamma(n) = (n-1)!$	$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
t-Score of a Sample	$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$	
Conditions	• Is typically used: 1. With small sample sizes or 2. When the population standard deviation is unknown • Similar in shape to the normal distribution. $Z \sim \mathcal{N}(0,1)$ • Used for inference about means (Use χ^2 for variance).	
Variables	x = observed value *df* = degrees of freedom = $n-1$	
TI-84	[2nd] [DISTR] 5:tpdf(x, df) $f(x) = P(X = v)$ [2nd] [DISTR] 6:tcdf(-∞, t, df) $F(x) = P(X \le v)$	

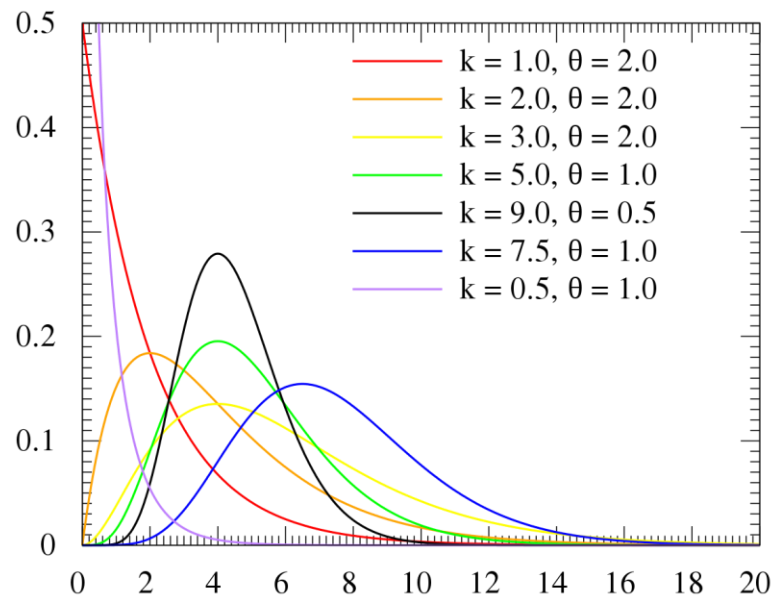
Python	`import scipy.stats as st`
	`mean, sd, t, df = 0, 1, -0.25, 30`
	
	`print(st.t.cdf(t, df, mean, sd))    # P(t <= -0.25)`
	`print(st.t.sf(t, df, mean, sd))     # P(t >= 1.5)`
	`                                    # P(49 < t < 60)`
	`print(st.t.cdf(60, df, mean, sd) - st.t.cdf(49, df, mean, sd))`
	`print(st.t.ppf(0.135, df, mean, sd))  # P(t < t*) = p = 0.135`
	`print(st.t.isf(0.405, df, mean, sd))  # P(t > t*) = p = 0.405`

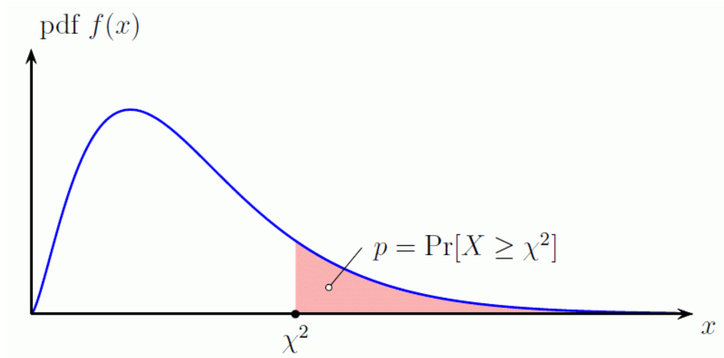
Example	*Suppose scores on an IQ test are normally distributed, with a population mean of 100. Suppose 20 people are randomly selected and tested. The standard deviation in the sample group is 15. What is the probability that the average test score in the sample group will be at most 110?* Solution: n=20, df=20-1=19, μ = 100, $\bar{x}$=110, s = 15 tcdf(-1E99, (110-100)/(15/sqrt(20)), 19) = 0.996 = ~99.6%
Online PDF Calculator	http://keisan.casio.com/exec/system/1180573204

Student's t Distribution Table:

Cum. Prob.	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
1-tail α	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
2-tails α	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.0000	1.0000	1.3764	1.9626	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6192
2	0.0000	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991
3	0.0000	0.7649	0.9785	1.2498	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240
4	0.0000	0.7407	0.9410	1.1900	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103
5	0.0000	0.7267	0.9195	1.1558	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934	6.8688
6	0.0000	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588
7	0.0000	0.7111	0.8960	1.1192	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853	5.4079
8	0.0000	0.7064	0.8888	1.1081	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008	5.0413
9	0.0000	0.7027	0.8834	1.1000	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809
10	0.0000	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869
11	0.0000	0.6974	0.8755	1.0877	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247	4.4370
12	0.0000	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296	4.3178
13	0.0000	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2208
14	0.0000	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405
15	0.0000	0.6912	0.8662	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728
16	0.0000	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.0150
17	0.0000	0.689	0.8633	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651
18	0.0000	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216
19	0.0000	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794	3.8834
20	0.0000	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518	3.8495
21	0.0000	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193
22	0.0000	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7921
23	0.0000	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7676
24	0.0000	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454
25	0.0000	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251
26	0.0000	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066
27	0.0000	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896
28	0.0000	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739
29	0.0000	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594
30	0.0000	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460
40	0.0000	0.6807	0.8507	1.0500	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510
50	0.0000	0.6794	0.8489	1.0473	1.2987	1.6759	2.0086	2.4033	2.6778	3.2614	3.4960
60	0.0000	0.6786	0.8477	1.0455	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602
70	0.0000	0.6780	0.8468	1.0442	1.2938	1.6669	1.9944	2.3808	2.6479	3.2108	3.4350
80	0.0000	0.6776	0.8461	1.0432	1.2922	1.6641	1.9901	2.3739	2.6387	3.1953	3.4163
90	0.0000	0.6772	0.8456	1.0424	1.2910	1.6620	1.9867	2.3685	2.6316	3.1833	3.4019
100	0.0000	0.6770	0.8452	1.0418	1.2901	1.6602	1.9840	2.3642	2.6259	3.1737	3.3905
1000	0.0000	0.6747	0.8420	1.0370	1.2824	1.6464	1.9623	2.3301	2.5808	3.0984	3.3003
∞ → z	0.0000	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905
	0%	**50%**	**60%**	**70%**	**80%**	**90%**	**95%**	**98%**	**99%**	**99.8%**	**99.9%**
	Confidence Level C										

Probability Density Function (PDF)	Mean	Standard Deviation
F Distribution	 This distribution is also known as **Snedecor's *F* distribution** or the **Fisher–Snedecor distribution** (after Ronald Fisher and George W. Snedecor).	
Parameters	d_1, d_2 > 0 degrees of freedom	
$P(x) = \dfrac{\sqrt{\dfrac{(d_1 x)^{d_1}\, d_2^{\,d_2}}{(d_1 x + d_2)^{d_1+d_2}}}}{x\, B\left(\frac{d_1}{2}, \frac{d_2}{2}\right)}$	$\mu = \dfrac{d_2}{(d_2-2)}$ $for\ d_2 > 2$	$\sigma = \dfrac{d_2}{(d_2-2)}\sqrt{\dfrac{2(d_1+d_2-2)}{d_1(d_2-4)}}$ $for\ d_2 > 4$
Conditions	• The F-distribution with d_1 and d_2 degrees of freedom is the distribution of $X = \dfrac{S_1/d_1}{S_2/d_2}$ where S_1 and S_2 are independent random variables with chi-square distributions with respective degrees of freedom d_1 and d_2. • Arises frequently as the null distribution of a test statistic, most notably in the analysis of variance (ANOVA) and other *F*-tests.	
Variables	x = observed value	
TI-84	[2nd] [DISTR] 9: fpdf(x, ν_{num}, ν_{denom}) $f(x) = P(X = x)$ [2nd] [DISTR] 0: fcdf(-∞, t, ν_{num}, ν_{denom}) $F(x) = P(X \le x)$	
Example		
Online PDF Calculator	https://stattrek.com/online-calculator/f-distribution.aspx	

Probability Density Function (PDF)	Mean	Standard Deviation
Gamma Distribution	 The gamma function is the continuous version of the discrete factorial function, n!.	
Parameters	k > 0 shape θ > 0 scale	
$f(x) = \frac{1}{\Gamma(k)\,\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$	$\mu = k\theta$	$\sigma = \sqrt{k}\,\theta$
Where the Gamma function $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$	$\Gamma(n) = (n-1)!$	Where the Gamma function $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$
Conditions	• The exponential distribution, Erlang distribution, and chi-square distribution are special cases of the gamma distribution. • The gamma distribution is the maximum entropy probability distribution (both with respect to a uniform base measure and with respect to a 1/*x* base measure) for a random variable *X* for which **E**[*X*] = *kϑ* = *α*/*β* is fixed and greater than zero, and **E**[ln(*X*)] = *ψ*(*k*) + ln(*ϑ*) = *ψ*(*α*) − ln(*β*) is fixed (*ψ* is the digamma function).	
Variables	x = observed value	
TI-84	• GAMFUNC (Gamma function) PRGM • GAMDSTR (Gamma distribution function) PRGM	
Example		
Online PDF Calculator	https://keisan.casio.com/exec/system/1180573217	

<table>
<tr><th>Probability Density Function (PDF)</th><th>Mean</th><th>Standard Deviation</th></tr>
<tr><td>Chi-Square Distribution</td><td colspan="2">

Skewed-right (above) have fewer values to the right, and median < mean.</td></tr>
<tr><td>$\chi^2(x,k) = \frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{\frac{-x}{2}}$</td><td>$\mu = E(X) = k$
$\mu = \sqrt{2}\Gamma \frac{\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}\right)}$
Mode = $\sqrt{k-1}$</td><td>$\sigma^2 = \sqrt{2k}$
$\sigma^2 = k - \frac{2\Gamma\left(\frac{k+1}{2}\right)^2}{\Gamma\left(\frac{k}{2}\right)^2}$</td></tr>
<tr><td>Conditions</td><td colspan="2">• Used for inference about variance in categorical distributions.
• Used when we want to test the independence, homogeneity, and "goodness of fit" to a distribution.
• Used for counted data.</td></tr>
<tr><td>Variables</td><td colspan="2">x = observed value
ν = df = degrees of freedom = $n - 1$</td></tr>
<tr><td>TI-84</td><td colspan="2">[2nd] [DISTR] 7: χ^2pdf(x, ν) $f(x) = P(X = x, k)$
[2nd] [DISTR] 8: χ^2cdf(x, ν) $F(x) = P(X \leq x, k)$</td></tr>
<tr><td>Example</td><td colspan="2">χ^2pdf() is only used to graph the function.</td></tr>
<tr><td>Online PDF Calculator</td><td colspan="2">https://stattrek.com/online-calculator/chi-square.aspx</td></tr>
</table>

<table>
<tr><td>Uniform</td><td rowspan="11">See
http://www4.ncsu.edu/~swu6/documents/A-probability-and-statistics-cheatsheet.pdf</td></tr>
<tr><td>Log-Normal</td></tr>
<tr><td>Multivariate Normal</td></tr>
<tr><td>F</td></tr>
<tr><td>Exponential</td></tr>
<tr><td>Gamma</td></tr>
<tr><td>Inverse Gamma</td></tr>
<tr><td>Dirichlet</td></tr>
<tr><td>Beta</td></tr>
<tr><td>Weibull</td></tr>
<tr><td>Pareto</td></tr>
</table>

Continuous Probability Distribution Functions

Cumulative Distribution Function (CDF)	Mean	Standard Deviation
$P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$	If $f(x) = \Phi(x)$ (the Normal PDF), then no exact solution is known. Use z-tables or web calculator (http://davidmlane.com/normal.html).	
$\int_{-\infty}^{\infty} f(x)\,dx = 1$	The area under the curve is always equal to exactly 1 (100% probability).	
Integral of PDF = CDF (Distribution)	$F(x) = \int_{-\infty}^{x} f(x)\,dx$	Use the density function $f(x)$, not the distribution function $F(x)$, to calculate $E(X), Var(X)$ and $\sigma(X)$.
Derivative of CDF = PDF (Density)	$f(x) = \frac{dF(x)}{dx}$	
Expected Value (Mean)	$E(X) = \int_{a}^{b} x\, f(x)\,dx$	
Needed to calculate Variance	$E(X^2) = \int_{a}^{b} x^2\, f(x)\,dx$	
Variance		$Var(X) = E(X^2) - E(X)^2$
Standard Deviation		$\sigma(X) = \sqrt{Var(X)}$

Discrete Distributions

	Notation[1]	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\text{Unif}\{a,\ldots,b\}$	$\begin{cases} 0 & x<a \\ \frac{\lfloor x\rfloor-a+1}{b-a} & a\le x\le b \\ 1 & x>b \end{cases}$	$\frac{I(a<x<b)}{b-a+1}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$\frac{e^{as}-e^{-(b+1)s}}{s(b-a)}$
Bernoulli	$\text{Bern}(p)$	$(1-p)^{1-x}$	$p^x(1-p)^{1-x}$	p	$p(1-p)$	$1-p+pe^s$
Binomial	$\text{Bin}(n,p)$	$I_{1-p}(n-x,x+1)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^s)^n$
Multinomial	$\text{Mult}(n,p)$		$\frac{n!}{x_1!\ldots x_k!}p_1^{x_1}\cdots p_k^{x_k} \quad \sum_{i=1}^{k}x_i=n$	np_i	$np_i(1-p_i)$	$\left(\sum_{i=0}^{k}p_ie^{s_i}\right)^n$
Hypergeometric	$\text{Hyp}(N,m,n)$	$\approx\Phi\left(\frac{x-np}{\sqrt{np(1-p)}}\right)$	$\frac{\binom{m}{x}\binom{m-x}{n-x}}{\binom{N}{x}}$	$\frac{nm}{N}$	$\frac{nm(N-n)(N-m)}{N^2(N-1)}$	N/A
Negative Binomial	$\text{NBin}(n,p)$	$I_p(r,x+1)$	$\binom{x+r-1}{r-1}p^r(1-p)^x$	$r\frac{1-p}{p}$	$r\frac{1-p}{p^2}$	$\left(\frac{p}{1-(1-p)e^s}\right)^r$
Geometric	$\text{Geo}(p)$	$1-(1-p)^x \quad x\in\mathbb{N}^+$	$p(1-p)^{x-1} \quad x\in\mathbb{N}^+$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^s}$
Poisson	$\text{Po}(\lambda)$	$e^{-\lambda}\sum_{i=0}^{x}\frac{\lambda^i}{i!}$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$e^{\lambda(e^s-1)}$

http://www4.ncsu.edu/~swu6/documents/A-probability-and-statistics-cheatsheet.pdf

Continuous Distributions

	Notation	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\text{Unif}(a, b)$	$\begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x > b \end{cases}$	$\frac{I(a < x < b)}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{sb} - e^{sa}}{s(b-a)}$
Normal	$\mathcal{N}(\mu, \sigma^2)$	$\Phi(x) = \int_{-\infty}^{x} \phi(t)\, dt$	$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$	μ	σ^2	$\exp\left\{\mu s + \frac{\sigma^2 s^2}{2}\right\}$
Log-Normal	$\ln\mathcal{N}(\mu, \sigma^2)$	$\frac{1}{2} + \frac{1}{2}\text{erf}\left[\frac{\ln x - \mu}{\sqrt{2\sigma^2}}\right]$	$\frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\ln x - \mu)^2}{2\sigma^2}\right\}$	$e^{\mu + \sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$	
Multivariate Normal	$\text{MVN}(\mu, \Sigma)$		$(2\pi)^{-k/2} \lvert\Sigma\rvert^{-1/2} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$	μ	Σ	$\exp\left\{\mu^T s + \frac{1}{2} s^T \Sigma s\right\}$
Student's t	$\text{Student}(\nu)$	$I_x\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}$	0	0	
Chi-square	χ^2_k	$\frac{1}{\Gamma(k/2)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$	$\frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2} e^{-x/2}$	k	$2k$	$(1-2s)^{-k/2}\ s < 1/2$
F	$\text{F}(d_1, d_2)$	$I_{\frac{d_1 x}{d_1 x + d_2}}\left(\frac{d_1}{2}, \frac{d_1}{2}\right)$	$\frac{\sqrt{\frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}}{x\text{B}\left(\frac{d_1}{2}, \frac{d_1}{2}\right)}$	$\frac{d_2}{d_2 - 2}$	$\frac{2d_2^2(d_1 + d_2 - 2)}{d_1(d_2-2)^2(d_2-4)}$	
Exponential	$\text{Exp}(\beta)$	$1 - e^{-x/\beta}$	$\frac{1}{\beta} e^{-x/\beta}$	β	β^2	$\frac{1}{1-\beta s}\ (s < 1/\beta)$
Gamma	$\text{Gamma}(\alpha, \beta)$	$\frac{\gamma(\alpha, x/\beta)}{\Gamma(\alpha)}$	$\frac{1}{\Gamma(\alpha)\,\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta s}\right)^\alpha\ (s < 1/\beta)$
Inverse Gamma	$\text{InvGamma}(\alpha, \beta)$	$\frac{\Gamma\left(\alpha, \frac{\beta}{x}\right)}{\Gamma(\alpha)}$	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x}$	$\frac{\beta}{\alpha-1}\ \alpha > 1$	$\frac{\beta^2}{(\alpha-1)^2(\alpha-2)^2}\ \alpha > 2$	$\frac{2(-\beta s)^{\alpha/2}}{\Gamma(\alpha)} K_\alpha\left(\sqrt{-4\beta s}\right)$
Dirichlet	$\text{Dir}(\alpha)$		$\frac{\Gamma\left(\sum_{i=1}^k \alpha_i\right)}{\prod_{i=1}^k \Gamma(\alpha_i)} \prod_{i=1}^k x_i^{\alpha_i - 1}$	$\frac{\alpha_i}{\sum_{i=1}^k \alpha_i}$	$\frac{\mathbb{E}[X_i](1 - \mathbb{E}[X_i])}{\sum_{i=1}^k \alpha_i + 1}$	
Beta	$\text{Beta}(\alpha, \beta)$	$I_x(\alpha, \beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$1 + \sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r}\right)\frac{s^k}{k!}$
Weibull	$\text{Weibull}(\lambda, k)$	$1 - e^{-(x/\lambda)^k}$	$\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$	$\lambda\Gamma\left(1 + \frac{1}{k}\right)$	$\lambda^2\Gamma\left(1 + \frac{2}{k}\right) - \mu^2$	$\sum_{n=0}^{\infty} \frac{s^n \lambda^n}{n!} \Gamma\left(1 + \frac{n}{k}\right)$
Pareto	$\text{Pareto}(x_m, \alpha)$	$1 - \left(\frac{x_m}{x}\right)^\alpha\ x \geq x_m$	$\alpha\frac{x_m^\alpha}{x^{\alpha+1}}\quad x \geq x_m$	$\frac{\alpha x_m}{\alpha-1}\ \alpha > 1$	$\frac{x_m^\alpha}{(\alpha-1)^2(\alpha-2)}\ \alpha > 2$	$\alpha(-x_m s)^\alpha \Gamma(-\alpha, -x_m s)\ s < 0$