**Harold’s Sets Cheat Sheet**

26 October 2022

**Set Definitions**

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| **Term** | **Definition** | **Examples** |
| **Set** | A well-defined collection of distinct mathematical objects | C = {2, 4, 5} denotes a set of three numbers: 2, 4, and 5  D = {(2, 4), (−1, 5)} denotes a set of two ordered pairs of numbers |
| **Element** | Objects, members | a, 3, (x, y) |
| **Pair** | Ordered pair.  An element with two members.  Order matters. | (x, y) |
| **Tuple** | Ordered tuple.  A column of three mathematical objects.  Order matters. |  |
| **n-Tuple** | Ordered n-tuple.  Is the set of all **3-tuples** whose entries are integers.  Order matters. |  |
| **Set-Builder Notation** | Set | The set of cubes of the first 100 positive integers. |
| **Roster Notation** | A list of the elements enclosed in curly braces with the individual elements separated by commas | A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} |

**Set-Builder Notation**

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| **Text  Description automatically generated** | **Text  Description automatically generated** |

Set-Builder Notation: { x ∈ ℝ | x ≤ 2 or x > 3 }

Number Line: two intervals

Interval Notation: (−∞, 2] U (3, +∞)

**Number Sets**

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| **Symbol** | **Definition** | **Set Notation** | **Examples** | **Equations** |
| ∅ | **Empty** or null set | { } | ∅ ∈ {∅} |  |
| ℕ | **Natural** numbers | {x ∈ ℤ : x > 0} | {1, 2, 3, …} or  {0, 1, 2, 3, …}  (per ISO 80000-2 2-6.1) |  |
| 𝕎 | **Whole numbers** | {x ∈ ℤ : x ≥ 0} | {0, 1, 2, 3, …} | n ≥ 0 |
| ℙ | **Prime** numbers | {a, b ∈ ℤ+ : (p∖ab ⟶ p∖a ∨ p∖b)} | {2, 3, 5, 7, 11, 13, ...} | unofficial |
| ℤ | **Integers** | {x : x = ±ℕ ∨ x = 0} | {…, −3, −2, −1, 0, 1, 2, 3, …} |  |
| ℚ | **Rational** numbers | {p/q : p, q ∈ ℤ ∧ q ≠ 0} | {0, ¼, ½, ¾, 1} |  |
| 𝕀 | **Irrational** numbers | {x ∈ ℝ : x ∉ ℚ} | {0, ¼, ½, ¾, 1} |  |
| 𝔸 | **Algebraic** numbers | { x ∈ ℝ : x = root of a one variable polynomial ∧ coefficients ∈ ℚ} | {5, -7, ½, } |  |
| 𝕋 | **Transcendental** numbers | {x ∈ ℝ : x ∉ 𝔸, x ∉ ℚ} | {π, e, eπ, sin(x), logb a} | 𝕋 = 𝕌 − 𝔸 |
| ℝ | **Real** numbers | {x : x corresponds to a number on the number line} | { π , 3.1415, -1, ⅞, } |  |
| 𝕀 | **Imaginary** numbers | {b : bi where } | {2i, } |  |
| ℂ | **Complex** numbers | {a, b ∈ ℝ : a + bi} | {1 + 2i, -3.4i, ⅝} |  |
| 𝕌 | **Universal** set | all possible values in a particular context | | |
| {0} | **Zero integer** | {x ∈ ℤ : x = 0} | {0} | n = 0 |
| ℤ − {0} | **Non-zero integers** | {x ∈ ℤ : x ≠ 0} | {…, −3, −2, −1, 1, 2, 3, …} | n ≠ 0 |
| ℤ+ | **Positive integers** | {x ∈ ℤ : x > 0} | {1, 2, 3, ...} | n > 0 |
| ℤ− | **Negative integers** | {x ∈ ℤ : x < 0} | {..., −3, −2, −1} | n < 0 |
| ℕ ⋃ {0} | **Non-negative integers** | {x ∈ ℤ : x ≥ 0} | {0, 1, 2, 3, …} | n ≥ 0 |
| ℤ− ⋃ {0} | **Non-positive integers** | {x ∈ ℤ : x ≤ 0} | {..., −3, −2, −1, 0} | n ≥ 0 |
| {0}, ℝ˟ | **Zero real** | {x ∈ ℝ : x = 0} | {0.0} | x = 0 |
| ℝ − {0}  ℝ \ {0} | **Non-zero real** numbers | {x ∈ ℝ : x ≠ 0} | {-0.001, 0.002} | x ≠ 0 |
| ℝ+  (0, ∞) | **Positive real** numbers | {x ∈ ℝ : x > 0} | {0.0001, 0.0002, ...} | x > 0 |
| ℝ−  (−∞, 0) | **Negative real** numbers | {x ∈ ℝ : x < 0} | {..., −0.0002, −0.0001} | x < 0 |
| [0, ∞) | **Non-negative real** numbers | {x ∈ ℝ : x ≥ 0} | {0, 0.0001, 0.0002, ...} | x ≥ 0 |
| (−∞, 0] | **Non-positive real** numbers | {x ∈ ℝ : x ≤ 0} | {..., −0.0002, −0.0001, 0} | x ≤ 0 |
|  | | | | |
| See the source image | | | | |

**Set Laws**

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| **Law** | **Union Example** | **Intersection Example** |
| **Idempotent Laws** | A ⋃ A = A | A ⋂ A = A |
| **Associative Laws** | (A ⋃ B) ⋃ C = A ⋃ (B ⋃ C) | (A ⋂ B) ⋂ C = A ⋂ (B ⋂ C) |
| **Commutative Laws** | A ⋃ B = B ⋃ A | A ⋂ B = B ⋂ A |
| **Distributive Laws** | A ⋃ (B ⋂ C) = (A ⋃ B) ⋂ (A ⋃ C) | A ⋂ (B ⋃ C) = (A ⋂ B) ⋃ (A ⋂ C) |
| **Identity Laws** | A ⋃ ∅ = A | A ⋂ 𝕌 = A |
| **Domination Laws** | A ⋃ 𝕌 = 𝕌 | A ⋂ ∅ = ∅ |
| **Double Complement Law** | (Ac)c = A | |
| **Complement Laws** | A ⋃ Ac = 𝕌 | A ⋂ Ac = ∅ |
| **Complements of 𝕌 and ∅** | 𝕌c = ∅ | ∅c = 𝕌 |
| **De Morgan’s Laws** | (A ⋃ B)c = Ac ⋂ Bc | (A ⋂ B)c = Ac ⋃ Bc |
| **Absorption Laws** | A ⋃ (A ⋂ B) = A | A ⋂ (A ⋃ B) = A |
| **Set Difference Law** |  | A \ B = A ⋂ Bc  A - B = A ⋂ Bc |

**Set Properties**

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| **Property** | **Description** | **Examples** |
| **Composition** | Objects may be of various types.  A set may contain elements of different varieties. | A = {2, strewberry, monkey} |
| **Order** | The order in which the elements are listed is unimportant | A = { 10, 6, 4, 2 } |
| **Duplicates** | Repeating an element does not change the set | A = { 2, 2, 4, 6, 10 } |
| **Notation** | Typically, capital letters will be used as variables denoting sets, and lower case letters will be used for elements in the set | A = {a, b} |
| **Range** | Every set A | ∅ ⊆ A ⊆ *U* |
| **Empty Set** | Set with no members. | ∅ is a subset of every set. |

**Set Notation**

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| **Term** | **Definition** | **Examples** |
| { }  ｛ ｝ | Denotes a **set** | A = {a, e, i, o, u} |
| |  : | ‘**Such tha**t’ or ‘for which’ | B = {x| x ∈ ℕ and x ≤ 5 }  B = {x: x ∈ ℕ and x ≤ 5 } |
| ⇒  ≡ | Is **equivalent** or identical to | (C ∩ E) ⇒ (x ∊ C ∧ x ∊ E) |
| |A| | **Cardinality** of A, the number of elements in set A | if A = {(1,2), (3,4), (5,6)},  then |A| = 3 |
| A = B | If and only if they have precisely the **same** elements. A is **equal** to b. | if A = {4, 9} and B = {n2 : n=2 or n=3}, then A = B |
| A ⊆ B | If and only if every element of A is also an element of B. A is a **subset** of B. | {1, 8, 1107} ⊆ ℕ |
| A ⊈ B | A is not a **subset** of B.  A is **not contained** in B. | {-1, -8, -1107} ⊈ ℕ |
| A ⊂ B | A is a **proper subset** of B.  A is a subset of B that is not equal to B. | {1, 8, 1107} ⊂ ℕ |
| A ⊄ B | A is not a **proper subset** of B.  A is **not contained** in B. | {-1, -8, -1107} ⊄ ℕ |
| B ⊇ A | If and only if every element of A is in B.  B is a **superset** of A. | {1, 8, 1107} ⊆ ℕ |
| a ∊ A  A ∈ B  a ∈ A | A is a member of, an **element** of, or in A | ¾ ∈ ℚ |
| a ∉ A | A is not a member of A, is **not** an **element** of A | 3.14 ∉ ℤ |
| A ∩ B  A ⋂ B  A ∩ B | The set containing elements that are in both A and B.  A∩B is the **intersection** of A and B. | if A = {1, 2} and B = {2 ,3},  then A ⋂ B = {2} |
| A ∪ B  A ⋃ B  A ∪ B | The set containing elements that are in either A or B or both.  A∪B is the **union** of A and B. | if A = {1, 2} and B = {2, 3},  then A ⋃ B = {1, 2, 3} |
| A ∖ B  A − B | **Set difference**. The set containing elements that are in A **but not** in B.  A∖B is “A drop B”. A−B is “A difference B”. | if A = {1, 2} and B = {2, 3},  then A ∖ B = {1} |
| A ⊕ B | **Symmetric difference** is the set of elements that are a member of exactly one of A and B, but not both | A ⊕ B = ( A - B ) ∪ ( B - A ) |
| A ⋂ B = ∅ | A and B are **disjoint** sets.  No elements in common. | A ⋂ B = ∅ |
| Ak | Cartisian product of a set A with itself | Ak = A x A x ... x A k times |

**Logical Form of Set Notation**

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| **Set Notation** | **Logical Statement** | **Description** |
| A | x ∈ A  ∀x {x ∈ A} | * Is an element of |
| ¬A | x ∉ A  ∀x {x ∉ A} | * Is not an element of |
| A = B  A ﹦ B | A ⟷ B  ∀x [(x ∈ A ⟶ x ∈ B) ∧ ( x ∈ B ⟶ x ∈ A)]  A ⊆ B ∧ B ⊆ A | * Equal * Equivalence * Iff * ≝ |
| A ≠ B  A ≠ B | ∀x (x ∈ A ∧ x ∉ B) | * Not equal |
| A ⊆ B | ∀x (x ∈ A ⟶ x ∈ B)  ∀x ∈ A (x ∈ B)  x ∉ A \ B | * Subset of * *A* ∩ *B = A* ⟶A ⊆ B |
| A ⊈ B | ∃x (x ∈ A ∧ x ∉ B) | * Not a subset of |
| A ∩ B | ∀x (x ∈ A ∧ x ∈ B) | * Intersection |
| A ⋃ B | ∀x (x ∈ A ∨ x ∈ B) | * Union |
| A \ B | ∀x (x ∈ A ∧ x ∉ B) | * Difference * But Not |
| A ⊕ B | ∀x { x ∈ A - B ∨ x ∈ B - A } | * Exactly one |
| A ⟶ B | ∀x (x ∉ A ∨ x ∈ B) | * If – Then |
| A ⋂ B = ∅ | ¬∃x (x ∈ A ∧ x ∈ B)  ∀x ¬ (x ∈ A ∧ x ∈ B)  ∀x (x ∉ A ∨ x ∉ B)  ∀x (x ∈ A ⟶ x ∉ B) | * A nd B are disjoint, having no elements in common |
| ℱ | {Ai | i ∈ I} | * Family of sets |
| x ∈ ⋂ℱ | {x | ∀A ∈ ℱ ( x ∈ A)}  {x | ∀A (A ∈ ℱ ⟶ x ∈ A)} | * Intersection of family of sets |
| x ∈ ⋃ℱ | {x | ∃A ∈ ℱ ( x ∈ A)}  {x | ∃A (A ∈ ℱ ∧ x ∈ A)} | * Union of family of sets |
| ⋂ℱ | ⋂i ∈ I Ai = {x | ∀i ∈ I (x ∈ Ai)}  ⋂i ∈ I Ai = A1 ⋂ A2 ⋂ A3 ⋂ A4 ⋂ ... | * Intersection of an indexed family of sets |
| ⋃ℱ | ⋃i ∈ I Ai = {x | ∃i ∈ I (x ∈ Ai)}  ⋃i ∈ I Ai = {x ∈ I | ∃i ∈ I A (i, x)}  ⋂i ∈ I Ai = A1 ⋃ A2 ⋃ A3 ⋃ A4 ⋃ ... | * Union of an indexed family of sets |
| x ∈ ℘(A) | x ⊆ A  ∀y (y ∈ *x* ⟶ y ∈ A) | * Power Set * All subsets of set A, including ∅ * |P(A)| = 2|A| |

**Logical Form of Numbers**

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| **Definition** | **Logical Statement** | **Description** |
| Even | ∃*k* ∈ ℤ (*x* = 2*k*)  Set *E* = {2*k* : *k* ∈ ℤ}  2ℤ | * Definition of Even |
| Odd | ∃*k* ∈ ℤ (*x* = 2*k* + 1)  Set *O* = {2*k* + 1 : *k* ∈ ℤ} | * Definition of Odd |
| Prime | ∀a, b ∈ ℤ+ | (p∖ab ⟶ p∖a  ∨  p∖b) | * A positive integer *p* > 1 that has no positive integer divisors other than 1 and *p* itself is prime. * Here \ means ”is a divisor of” |
| Not Prime | ∃a, b ∈ ℤ+ (ab = n ∧ a < n ∧ b < n) | * a and b are factors of n, so not prime |
| Divides | x | y ⟷∃k ∈ ℤ (y = kx) | * Divisability * Divides * Divides into * x divides y evenly * x | y to mean “x divides y,” * x ∤ y means “x does not divide y” |
| **Rational** | r ∈ ℝ ∃x, y ∈ ℤ ((y ≠ 0) ∧ (r = x/y)) ⟶ r ∈ ℚ | * Definition of Rational number * A fraction composed of two integers, but no division by 0 |

**Logical Form of Geometry**

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| **Definition** | **Logical Statement** | **Description** |
| **Line** | {(x, y) ∈ ℝ × ℝ | y = mx + b}  = {(0, b), (1, m + b), (2, 2m + b), …} | * You can think of the graph of the equation as a picture of its truth set! |
| **Plane** | ℝ × ℝ = {(x, y) | x and y are real numbers} | * These are the coordinates of all the points in the plane * ℝ2 = ℝ × ℝ |
| **3D Space** | ℝ3 = {(x, y, z) | x, y and z are real numbers} | * These are the coordinates of all the points in 3D space * ℝ3 = ℝ × ℝ × ℝ |
| **Spacetime** | ℝ4 = {(x, y, z, t) | x, y, z and t are real numbers} | * These are the coordinates of all the points in 3D space and 1D time * ℝ4 = ℝ × ℝ × ℝ × ℝ |

**Logical Form of Functions**

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| **Definition** | **Logical Statement** | **Description** |
| Function | *f* : *x* ⟶ *y*  ∀*x* ∈ X ∃!*y* ∈ Y ((*x*, *y*) ∈ *f*)  *f* = {(*a*, *b*) ∈ A × B | b = f(a)} | * Function * *f* is a relation frrom A to B * Example : *f* = {(x, y) ∈ ℝ × ℝ | y = x2} |
| Domain | Dom(*f* )  X | * Domain of *f* |
| Range | Ran(*f* )  {*f* (a) | a ∈ A}  Y | * Range ⊆ co-domain * Co-domain * Image of f (linear algebra term) |
| Surjection | *f* = ∀*y* ∈ Y {∃ at **least** one *x* ∈ X such that *f(x) = y*}  ∀*y* ∈ Y, ∃*x* ∈ X | (*f(x) = y*)  Ran(*f* ) = Y | * **Onto** * Surjective f * Every y is mapped to by at least one x * No orphan y’s * *e.g., y is dating at least one x* |
| Injection | *f* = ∀*y* ∈ Y {∃ at **most** one *x* ∈ X such that *f(x) = y*}  ¬∃a1 ∈ A ∃a2 ∈ A (*f* (a1) = *f* (a2) ∧ a1 ≠ a2)  **∀a1, a2 ∈ A | (*f* (a1) = *f* (a2) ⟶ a1 = a2)**  f(x) = f(y) ⟷ x = y  f(x) ≠ f(y) ⟷ x ≠ y | * **One-to-one** * Injective f * For any y there is at most one x * Can have orphan y’s * *e.g., y is either married or single* |
| Bijection | *f* = iff ∀*y* ∈ Y {∃ a **unique** *x* ∈ X such that *f(x) = y*} | * Bijective = surjective and injective * One-to-one correspondence * Bijective f * Invertible f * Iff has a well-defined inverse (*f-1*) * Iff both surjective and injective * One-to-one and onto * *e.g., Everyone is married to a spouse* |
| Inverse | *f−1: B* ⟶ *A*  ∀*b ∈* B∃!*a ∈* A *((b, a) ∈ f−1)*  *f(g(x)) = x*  f−1 ◦ f = iA and f ◦ f−1 = iB | * Inverse f |
| k-to-1 **Correspondence** | *Let X and Y be finite sets. The function f:X→Y is a k-to-1 correspondence if for every y ∈ Y, there are exactly k different x ∈ X such that f(x) = y.* | * Bijection is k = 1 |

**Cartesian Product**

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| **Set Notation** | **Logical Statement** | **Description** |
| A × B  A ⨯ B  A × B  A × B | {(a, b) | a ∈ A ∧ b ∈ B}  {(a, b) | a ∈ A, b ∈ B} | * Cartesian product * Cross product * Set of all ordered pairs in which the first entry is in A and the second entry is in B |

**Properties of Cartesian Products**

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| **Law** | **Logical Statement** | **Description** |
| **Distributive** | A × (B ⋂ C) = (A × B) ⋂ (A × C) | * ×⋂ |
| A × (B ⋃ C) = (A × B) ⋃ (A × C) | * ×⋃ |
| **Commutative** | (A × B) ⋂ (C × D) = (A ⋂ C) × (B ⋂ D) | * ×⋂× |
| ​(A × B) ⋃ (C × D) ⊆ (A ⋃ C) × (B ⋃ D) | * ×⋃× |
| **Domination** | ​A × ∅ = ∅  ∅ × A = ∅ | * ×∅ |

**Relations**

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| **Property** | **iA** | **Equivalence**  **(=)** | **Partial Order**  **(Poset)** | **Total Order**  **(Linear)** |
| **Reflexive** | ✓ | ✓ | ✓ |  |
| **Symmetric** | ✓ | ✓ |  |  |
| **Anti-Symmetric** |  |  | ✓ | ✓ |
| **Asymmetric** |  |  |  |  |
| **Transitive** | ✓ | ✓ | ✓ | ✓ |
| **Total** |  |  |  | ✓ |
| **Density** |  |  |  |  |
| **Binary Relation** |  | ✓ |  |  |

**Set Relations (xRy)**

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| **Set Notation** | **Logical Statement** | **Description** |
| **Relation** | R ⊆ A × B  ∀x (x ∈ R ⟶ x ∈ A × B)  R = {(a, b) ∈ A × B | conditions}  xRy = (x, y) ∈ R  Example: Dr = {(x, y) ∈ ℝ × ℝ | x and y differ by less than r} ⇒ |x − y| < r} | * Relation from A to B * R is a subset of the cross product |
| **Domain** | Dom(R)  {a ∈ A | ∃b ∈ B ((a, b) ∈ R)}  Dom(A) ⊆ A | * The domain of R is the set containing all the first coordinates of its ordered pairs |
| **Codomain**  **(Target)** |  | * All possible values in the range set * Ran(R) is a subset of the Target * The set of the possible output values of a function * The definition of a function |
| **Range**  **(Image)** | Ran(R)  {b ∈ B | ∃a ∈ A ((a, b) ∈ R)}  Ran(B) ⊆ B | * The range of R is the set containing all the second coordinates of its ordered pairs * The actual or most accurate output values of a function * The image of a function |
| **Inverse**  **(R−1)** | {(y, x) ∈ Y × X | (x, y) ∈ R}  (y, x) ∈ R−1 ⟷ (x, y) ∈ R  (x, y) ∈ R−1 ⟶ (x, y) ∈ R | * The inverse of R is the relation R−1 from B to A with the order of the coordinates of each pair reversed |
| **Composition**  **(S ∘ R)** | S ◦ R = (a, c) ∈ S ◦ R ⟷ ∃b | (a, b) ∈ R and (b, c) ∈ S  {(a, c) ∈ A × C | ∃b ∈ B ((a, b) ∈ R and (b, c) ∈ S)}  aRb and bSc  {(a, c) ∈ A × C | ∃b ∈ B (aRb ∧ bSc)} | * The composition of S and R is the relation S ◦ R from A to C * aRb and bSc, meaning R:a ⟶ R:b ⟶ S:b ⟶ S:c, so (R:a, S:c) * Ring operator |
| **Identity**  **(iA)** | {(x, y) ∈ A × A | x = y}  {(x, x) | x ∈ A} | * Identity relation |

**Order Properties of Binary Relations with Two Sets**

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| **Property** | **Logical Statement** | **Description** |
| **Reflexive** | xRx  (x, x) ∈ R  ∀x ∈ A (xRx)  ∀x ∈ A ((x, x) ∈ R) | * iA ⊆ R   where iA is the identity relation of set A or iA = {(x, x) | x ∈ A}   * **Directed graph**: Loop |
| **Anti-Reflexive** | ¬ (xRx)  ∀x ∈ A ¬ (xRx) | * **Directed graph**: No loops |
| **Symmetric** | xRy ⟶ yRx  ∀x ∈ A ∀y ∈ A (xRy ⟶ yRx) | * R = R-1 * **Directed graph**: 2-way arrow (edges come in pairs) or no arrows |
| **Anti-Symmetric** | (xRy ∧ yRx) ⟶ (x = y)  (x ≠ y) ⟶ ¬ (xRy) ∨ ¬ (yRx)  ∀x ∈ A ∀y ∈ A ((xRy ∧ yRx) ⟶ (x = y)) | * Equivalence * **Directed graph**: An arrow from x to y implies that there is no arrow from y to x |
| **Asymmetric** | xRy ⟶ ¬ (yRx)  ∀x ∈ A ∀y ∈ A ∀z ∈ A (xRy ⟶ ¬ (yRx)) | * Fails the vertical line test, so not a proper function, f(x) * **Directed graph**: 1-way arrow |
| **Transitive** | (xRy ∧ yRz) ⟶ xRz  ∀x ∀y ∀z ((xRy ∧ yRz) ⟶ xRz)  ∀x ∈ A ∀y ∈ A ∀z ∈ A ((xRy ∧ yRz) ⟶ xRz) | * R ◦ R ⊆ R * Similar to S ◦ R * **Directed graph**: Two routes from every vertex A to every vertex B, 1-hop and 2-hops   See the source image |
| **Total** | xRy ∨ yRx  ∀x ∈ A ∀y ∈ A (xRy ∨ yRx) | * Either-or |
| **Density** | xRy ⟶ ∃z | xRz ∧ zRy  ∀x ∈ A ∀y (xRy) ⟶ ∃z | xRz ∧ zRy | * A middle-man exists |
| **Binary** | R-1 ◦ R = Relation on set A  R ◦ R-1 = Relation on set C | * Relation on set <set> * Binary relation on set <set> |
| **Identity** | iA = {(x, y) ∈ A × A | x = y}  iA = {(x, x) | x ∈ A} | * Similar to a diagonal matrix |

**Mathematical Number Sets 🡪 Computer Science Data Types**

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| **Symbol** | **Definition** | **C Data Type** | **C++ Data Type** |
| ∅ | **empty** set,  set with no members | void | |
| ℕ | **natural** numbers | enum  unsigned  unsigned char  unsigned short  unsigned int  unsigned long  unsigned long long | |
| ℤ | **integers** | char  short  int  long  long long | |
| ℚ | **rational** numbers | NA | std::ratio<1, 10> |
| ℝ | **real** numbers | float  double  long double | |
| 𝕀 | **imaginary** numbers | (see complex below)  double complex z1; im = cimag(z1); | (see complex below)  std::complex <double> z1;  im = std::imag(z1); |
| ℂ | **complex** numbers | #include <complex.h>  float complex  double complex  long double complex | #include <complex>  std::complex<float>  std::complex <double>  std::complex <long double> |

**Sources**:

* [SNHU MAT 470](https://www.snhu.edu/admission/academic-catalogs/coce-catalog#/courses/VydU8ZIYx) - Real Analysis, [The Real Numbers and Real Analysis](https://www.amazon.com/gp/product/0387721762/ref=ox_sc_act_title_4?smid=A1C79WJQJ5SBBJ&psc=1), Ethan D. Bloch, Springer New York, 2011.
* See also “Harold’s Logic Cheat Sheet”.
* <https://www.storyofmathematics.com/set-notation>
* <https://math24.net/set-identities.html>