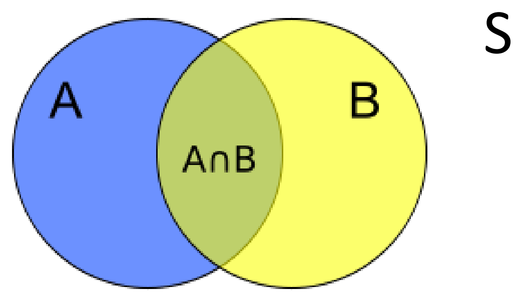
Harold's Probability
Cheat Sheet

22 October 2022

Probability

Rule	Formula	Definition
Notation	∩ = "and", Intersection, or "∧" ∪ = "or", Union, or "∨" $\overline{\square}$ = "not", negation, or "¬"	"and" implies multiplication. "or" implies addition. "not" implies negation.
Independent	If $P(A\|B) = P(A)$	The occurrence of one event does not affect the probability of the other, or vice versa.
Dependent	If $P(A \cap B) \neq \emptyset$	The occurrence of one event affects the probability of the other event.
Disjoint ("mutually exclusive")	If $P(A \cap B) = \emptyset$ Then $\boldsymbol{P(A \cup B) = P(A) + P(B)}$	The events can never occur together.
Probability ("likelihood")	$\boldsymbol{0 \leq P(E) \leq 1}$ $P(E) = \frac{\#\ Events\ (E)}{Sample\ Space\ (S)} = \frac{\#\ of\ Favorable\ Outcomes}{Total\ \#\ of\ Possible\ Outcomes}$	
Addition Rule ("or")	$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$	

Multiplication Rule
("and")
	if independent or disjoint:
	$\boldsymbol{P(A \cap B) = P(A)\ P(B)}$
	$\boldsymbol{P(A \cap B \cap C) = P(A)\ P(B)\ P(C)}$
	
	if dependent:
	$\boldsymbol{P(A \cap B) = P(A)\ P(B\|A)}$
	$P(A \cap B) = P(B)\ P(A\|B)$
	$P(A \cap B) = P(A) - P(A \cap \overline{B})$	

Complement Rule / Subtraction Rule ("not")	$\boldsymbol{P(S)} = P(A \cup \overline{A}) =$ $P(A) + P(\overline{A}) = \boldsymbol{1}$ $P(A) = 1 - P(\overline{A})$ $\boldsymbol{P(\overline{A}) = 1 - P(A)}$ $P(A\|B) + P(\overline{A}\|B) = 1$	The complement of event A (denoted $\overline{A}\ or\ A^c$) means "**not A**"; it consists of all simple outcomes that are not in A.
Conditional Probability ("given that")	$\boldsymbol{P(A\|B) = \frac{P(A \cap B)}{P(B)}}$ if independent or disjoint: $P(A\|B) = P(A)$ $P(B\|A) = P(B)$	Means the probability of event A given that event B occurred. Is a rephrasing of the Multiplication Rule. P(A|B) is the proportion of elements in B that are ALSO in A.

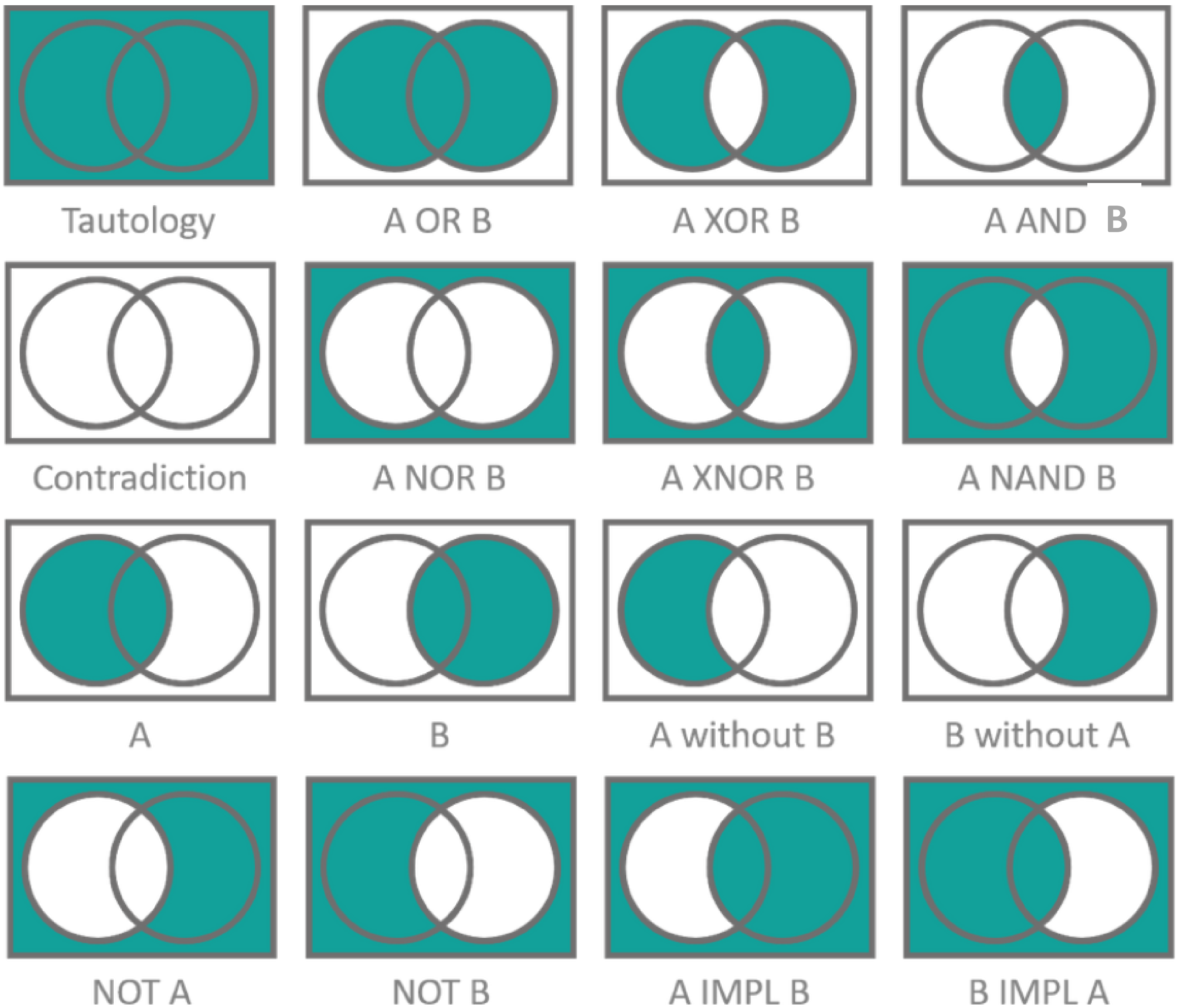
Total Probability Rule	$P(A) = P(A \cap B_1) + \cdots + P(A \cap B_n) = P(B_1)\, P(A\|B_1) + \cdots + P(B_n)\, P(A\|B_n)$ $P(A) = P(A \cap B) + P(A \cap \bar{B}) = P(B)\, P(A\|B) + P(\bar{B})\, P(A\|\bar{B})$	To find the probability of event A, partition the sample space into several disjoint events. A must occur along with one and only one of the disjoint events.
Bayes' Theorem	$\boldsymbol{P(A\|B)} = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\, P(B\|A)}{P(B)}$ $= \frac{\boldsymbol{P(A)\, P(B\|A)}}{\boldsymbol{P(A)\, P(B\|A) + P(\bar{A})\, P(B\|\bar{A})}}$	Allows P(A|B) to be calculated from P(B|A). Meaning it allows us to reverse the order of a conditional probability statement, and is the only generally valid method!
De Morgan's Law	$\overline{P(A \cup B)} \equiv \overline{P(A)} \cap \overline{P(B)}$ $\overline{P(A \cap B)} \equiv \overline{P(A)} \cup \overline{P(B)}$	Uses negation to convert an "or" to an "and". Uses negation to convert an "and" to an "or".

Discrete Distributions

Distribution	Formula
Probability Distribution	$\sum_{s \in S} p(s) = 1$
Factorial	$n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 3 \bullet 2 \bullet 1$
Permutation	$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$
Combination	$C(n,r) = {}_nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$
Uniform Discrete Distribution	$P(X = x) = \frac{1}{b-a+1}$ $P(S = s) = \frac{1}{\|S\|}$ *per outcome* $P(S = E) = \frac{\|E\|}{\|S\|}$ *per event*
Binomial Distribution	$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$
Geometric Distribution	$P(X \leq x) = q^{x-1}p = (1-p)^{x-1}p$ $P(X > x) = q^x = (1-p)^x$
Poisson Distribution	$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0,1,2,3,4, \ldots$
Bernoulli Distribution	$P(k;p) = p^k(1-p)^{1-k}\ for\ k\ \in \{0,1\}$
Trinomial Distribution	$P(X = x, Y = y) = \frac{n!}{x!\,y!\,(n-x-y)!} p_1^x p_2^y (1-p_1-p_2)^{n-x-y}$

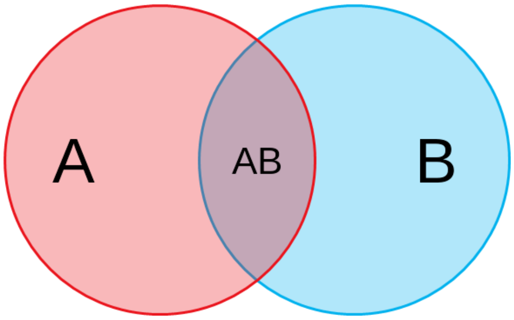
Hypergeometric Distribution	$P(x \mid N, m, n) = \frac{\left(\binom{m}{x}\binom{N-m}{n-x}\right)}{\binom{N}{n}}$
Negative Binomial Distribution	$P(X = r) = {}_{n+r-1}C_{r-1}\, p^r\, q^n$

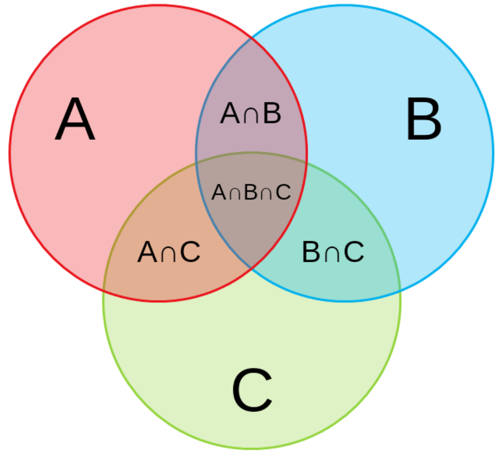
Venn Diagrams

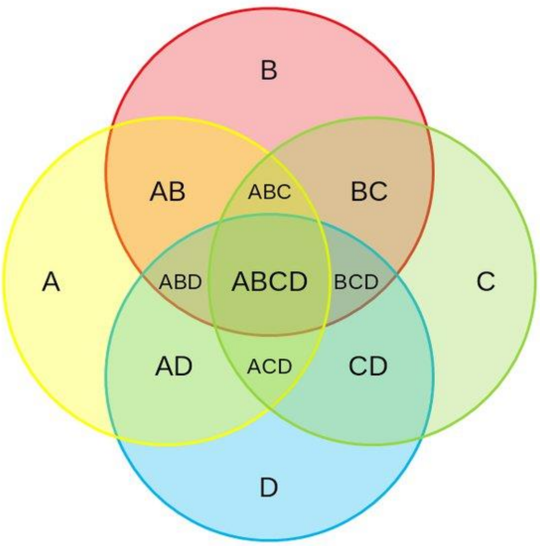


Sources:

- SNHU MAT 229 - Mathematical Proof and Problem Solving, How To Prove It - A Structured Approach, 3rd Edition - Daniel J. Vellman, Cambridge University Press, 2019.
- SNHU MAT 230 - Discrete Mathematics, zyBooks.


A
AB
B


A
A∩B
B
A∩B∩C
A∩C
B∩C
C


B
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ABC
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A
ABD
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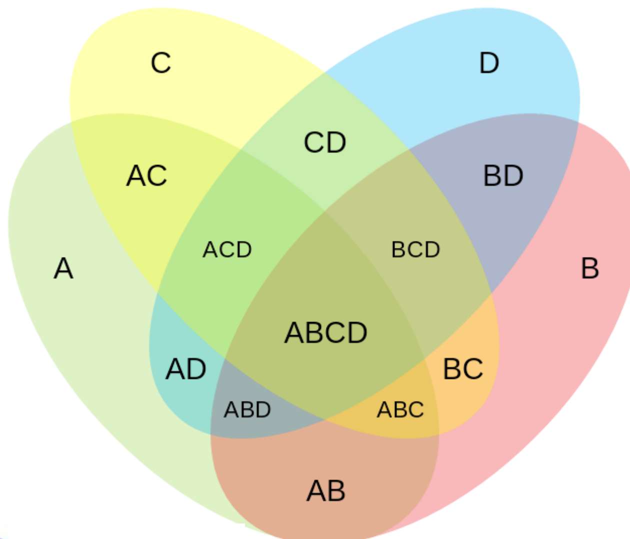
BCD
C
AD
ACD
CD
D


C
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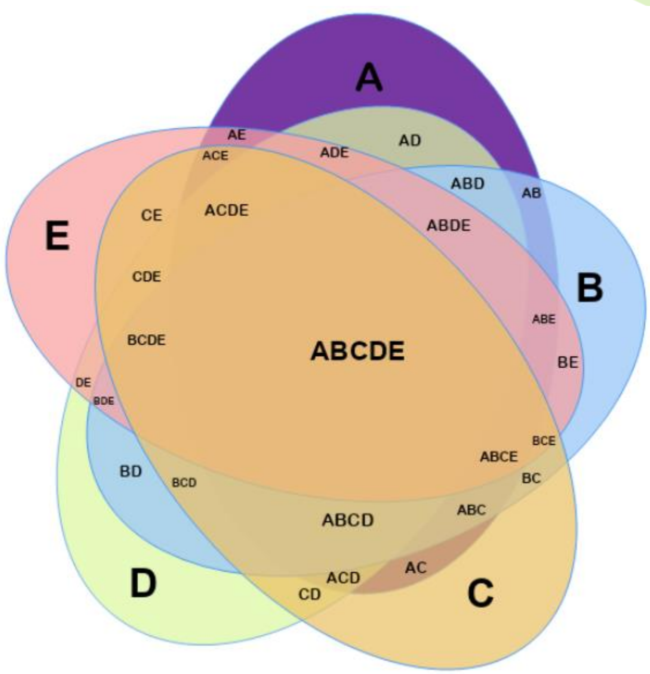
A
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C