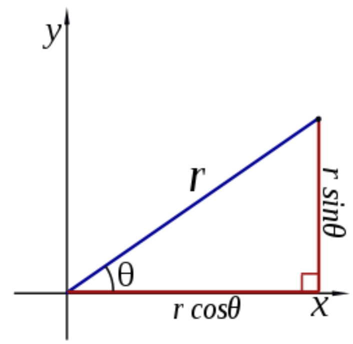
Harold's Precalculus
Cheat Sheet
29 November 2022

	Rectangular	Polar	Parametric
Point	$f(x) = y$ (x, y) (a, b)	(r, θ) or $r \angle \theta$ *Polar → Rect.* $x = r\cos\theta$ $y = r\sin\theta$ $\tan\theta = \frac{y}{x}$ *Rect. → Polar* $r^2 = x^2 + y^2$ $r = \pm\sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$	*Point (a,b) in Rectangular:* $x(t) = a$ $y(t) = b$ $< a, b >$ $t = 3^{rd}$ *variable, usually time, with 1 degree of freedom (df)*
Line	*Slope-Intercept Form:* $y = mx + b$ *Point-Slope Form:* $y - y_0 = m\,(x - x_0)$ *General Form:* $Ax + By + C = 0$ *Calculus Form:* $f(x) = f'(a)\,x + f(0)$		$< x, y > = < x_0, y_0 > + t < a, b >$ $< x, y > = < x_0 + at, y_0 + bt >$ where $< a, b > = < x_2 - x_1, y_2 - y_1 >$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$
Plane	$n_x(x - x_0)$ $+ n_y(y - y_0)$ $+ n_z(z - z_0) = 0$	*Vector Form:* $\boldsymbol{n} \bullet (\boldsymbol{r} - \boldsymbol{r}_0) = 0$	$\boldsymbol{r} = \boldsymbol{r}_0 + s\boldsymbol{v} + t\boldsymbol{w}$

<table>
<tr><th></th><th>Rectangular</th><th>Polar</th><th>Parametric</th></tr>
<tr><td>Conics</td><td>

General Equation for All Conics:

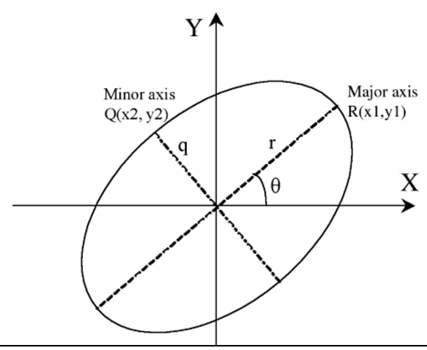
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

where
Line: $A = B = C = 0$
Circle: $A = C$ *and* $B = 0$
Ellipse: $AC > 0$
or $B^2 - 4AC < 0$
Parabola: $AC = 0$
or $B^2 - 4AC = 0$
Hyperbola: $AC < 0$
or $B^2 - 4AC > 0$
Note: If $A + C = 0$, *square hyperbola*

Rotation:
If B ≠ 0, then rotate coordinate system:

$\cot 2\theta = \frac{A - C}{B}$
$x = x' \cos\theta - y' \sin\theta$
$y = y' \cos\theta + x' \sin\theta$

New = (x’, y’), Old = (x, y)
rotates through angle θ from x-axis



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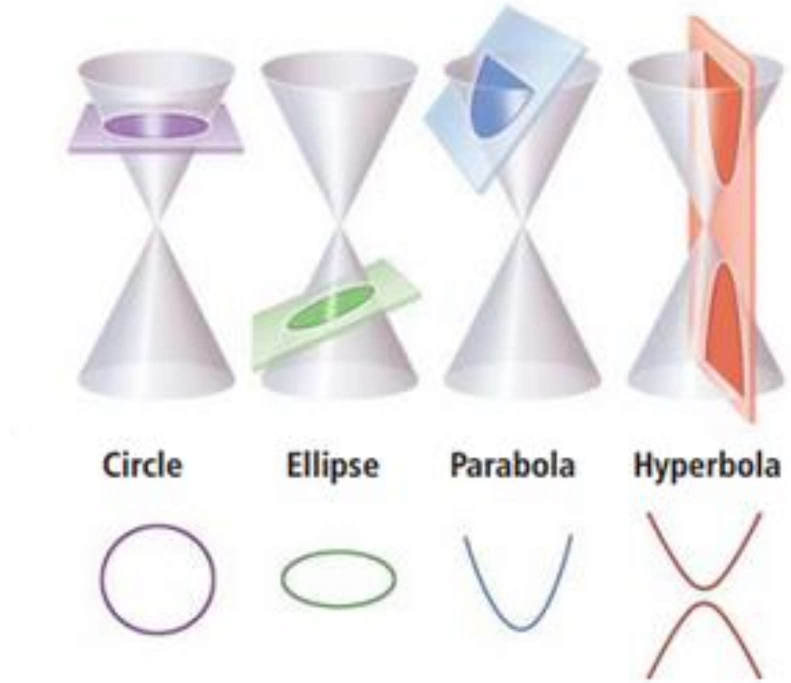

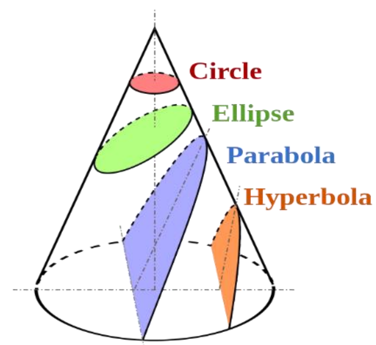

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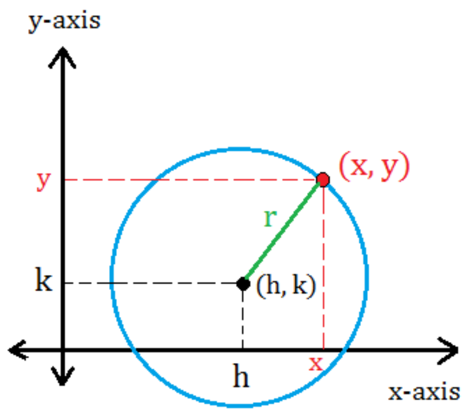
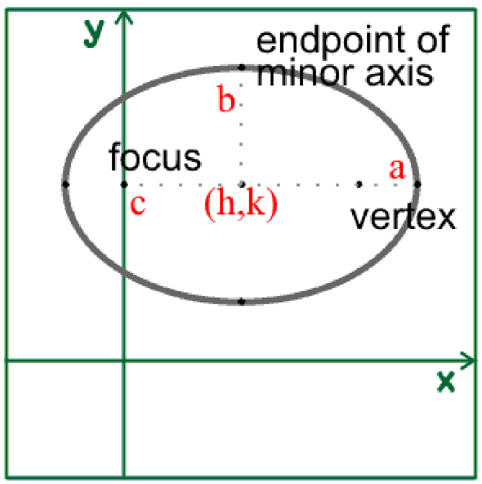
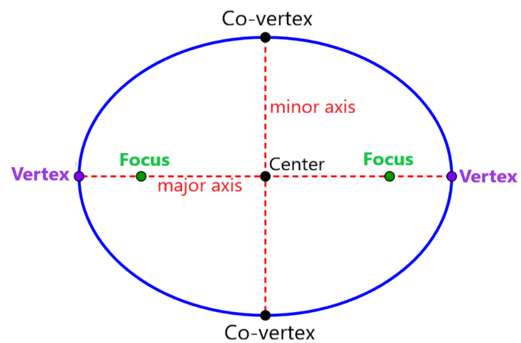
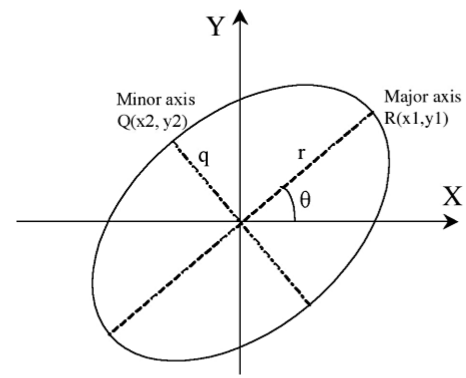
	Rectangular	Polar	Parametric
Circle	$(x-h)^2+(y-k)^2=r^2$ *Center:* (h,k) *Vertices: NA* *Focus:* (h,k)  	*Centered at Origin:* *r = a (constant)* $\theta\ =\ \theta\ [0,2\pi]\ or\ [0,360°]$	$x(t)=r\,cos(t)+h$ $y(t)=r\,sin(t)+k$ $[t_{min},\ t_{max}]=[0,2\pi)$ $(h,k)=center\ of\ circle$

	Rectangular	Polar	Parametric
Ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ *Center:* (h,k) *Vertices:* $(h\pm a,k)$ *Co-Vertices:* $(h,k\pm b)$ *Foci:* $(h\pm c,k)$ *Focus length, c, from center:* $c^2 = a^2 - b^2$ 	 ***Interesting Note:*** *The* <u>*sum*</u> *of the distances from each focus to a point on the curve is constant.* $\lvert d_1+d_2\rvert = k$	$x(t) = a\cos(t) + h$ $y(t) = b\sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi]$ $(h,k) = center\ of\ ellipse$ *Rotated Ellipse:* $x(t) = a\cos t\cos\theta - b\sin t\sin\theta + h$ $y(t) = a\cos t\sin\theta + b\sin t\cos\theta + k$ *θ = the angle between the x-axis and the major axis of the ellipse* 

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<tr>
<td>Hyperbola</td>
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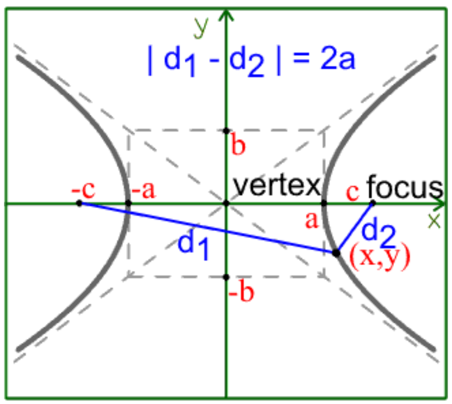
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center: (h, k)
Vertices: $(h \pm a, k)$
Foci: $(h \pm c, k)$

Focus length, c, from center:
$c^2 = a^2 + b^2$



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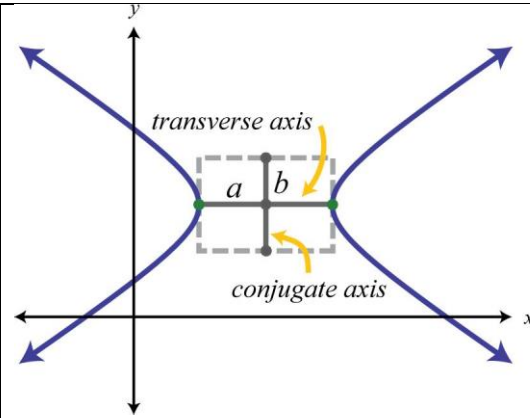

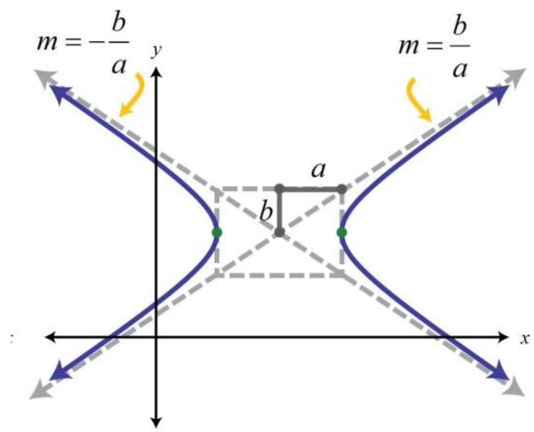

Interesting Note:
The <u>difference</u> between the distances from each focus to a point on the curve is constant.
$|d_1 - d_2| = k$

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Left-Right Opening Hyperbola:
$x(t) = a\,sec(t) + h$
$y(t) = b\,tan(t) + k$
$[t_{min}, t_{max}] = [-c, c]$
(h, k) = vertex of hyperbola

Up-Down Opening Hyperbola:
$x(t) = a\,tan(t) + h$
$y(t) = b\,sec(t) + k$
$[t_{min}, t_{max}] = [-c, c]$
(h, k) = vertex of hyperbola

General Form:
$x(t) = At^2 + Bt + C$
$y(t) = Dt^2 + Et + F$
where A and D have different signs

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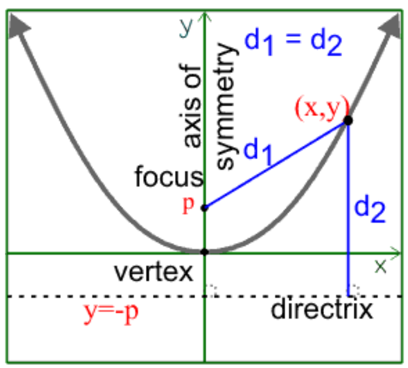
<tr><td>Parabola</td>
<td>Vertical Axis of Symmetry:
$x^2 = 4\,py$
$(x-h)^2 = 4p(y-k)$
Vertex: (h,k)
Focus: $(h,k+p)$
Directrix: $y = k-p$

Horizontal Axis of Symmetry:
$y^2 = 4\,px$
$(y-k)^2 = 4p(x-h)$
Vertex: (h,k)
Focus: $(h+p,k)$
Directrix: $x = h-p$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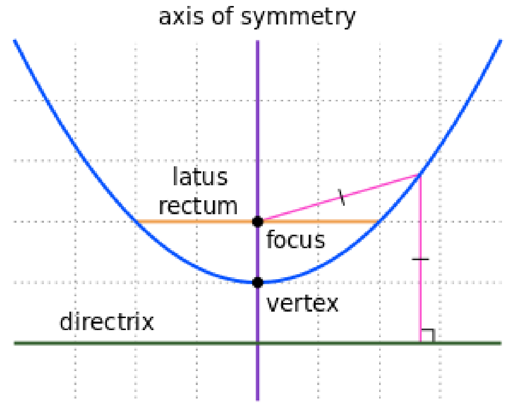
<td>

Vertical Axis of Symmetry:
$x(t) = 2pty = Dt^2 + Et + F$
where A and D have the same sign

Interesting Note:
The distances from a point on the curve to the focus is the <u>same</u> as to the directrix.</td>
<td>Vertical Axis of Symmetry:
$x(t) = 2pt + h$
$y(t) = pt^2 + k$ (opens upwards) or
$y(t) = -pt^2 - k$ (opens downwards)
$[t_{min}, t_{max}] = [-c, c]$
Vertex: (h,k)

Horizontal Axis of Symmetry:
$y(t) = 2pt + k$
$x(t) = pt^2 + h$ (opens to the right) or
$x(t) = -pt^2 - h$ (opens to the left)
$[t_{min}, t_{max}] = [-c, c]$
Vertex: (h,k)

Projectile Motion:
$x(t) = x_0 + v_x t$
$y(t) = y_0 + v_y t - 16t^2$ feet
$y(t) = y_0 + v_y t - 4.9t^2$ meters
$v_x = v\cos\theta$
$v_y = v\sin\theta$

General Form:
$x = At^2 + Bt + C$
$y = Dt^2 + Et + F$
where A and D have the same sign</td></tr>
<tr><td>Inverse Functions</td>
<td>$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Inverse Function Theorem:
$f^{-1}(b) = \dfrac{1}{f'(a)}$
where $b = f'(a)$</td>
<td>if $y = \sin\theta$ then $\theta = \sin^{-1} y$
if $y = \cos\theta$ then $\theta = \cos^{-1} y$
if $y = \tan\theta$ then $\theta = \tan^{-1} y$

if $y = \csc\theta$ then $\theta = \csc^{-1} y$
if $y = \sec\theta$ then $\theta = \sec^{-1} y$
if $y = \cot\theta$ then $\theta = \cot^{-1} y$</td>
<td>or $\theta = \arcsin y$
or $\theta = \arccos y$
or $\theta = \arctan y$

or $\theta = \text{arccsc}\, y$
or $\theta = \text{arcsec}\, y$
or $\theta = \text{arccot}\, y$</td></tr>
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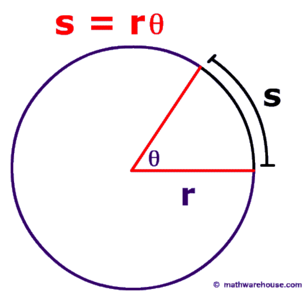
	Rectangular	Polar	Parametric
Arc Length		*Circle:* $L = s = r\theta$ *Proof:* $L = (fraction\ of\ circumference) \cdot \pi \cdot (diameter)$ $L = \left(\frac{\theta}{2\pi}\right)\pi\,(2r) = r\theta$	
Perimeter	*Square:* $P = 4s$ *Rectangle:* $P = 2l + 2w$ *Triangle:* $P = a + b + c$	*Circle:* $C = \pi d = 2\pi r$ *Ellipse:* $C \approx \pi(a + b)$	
Area	*Square:* $A = s^2$ *Rectangle:* $A = lw$ *Rhombus:* $A = ½\,ab$ *Parallelogram:* $A = Bh$ *Trapezoid:* $A = \frac{(B_1+B_2)}{2}\,h$ *Kite:* $A = \frac{d_1\,d_2}{2}$	*Triangle:* $A = ½\,Bh$ *Triangle:* $A = ½\,ab\sin(C)$ *Triangle using Heron's Formula:* $A = \sqrt{s(s-a)(s-b)(s-c)}$ $where\ s = \frac{a+b+c}{2}$ *Equilateral Triangle:* $A = \frac{\sqrt{3}}{4}s^2$	*Frustum:* $A = \frac{1}{3}\left(\frac{B_1+B_2}{2}\right)h$ *Circle:* $A = \pi r^2$ *Circular Sector:* $A = ½\,r^2\theta$ *Ellipse:* $A = \pi ab$
Lateral Surface Area	*Cylinder:* $SA = 2\pi rh$ *Cone:* $SA = \pi rl$		
Total Surface Area	*Cube:* $SA = 6s^2$ *Rectangular Box:* $SA = 2lw + 2wh + 2hl$ *Regular Tetrahedron:* $SA = 2bh$ *Cylinder:* $SA = 2\pi r\,(r + h)$	*Cone:* $SA = \pi r^2 + \pi rl = \pi r\,(r + l)$ *Sphere:* $SA = 4\pi r^2$ *Ellipsoid:* SA = (too complex)	
Volume	*Cube:* $V = s^3$ *Rectangular Prism:* $V = lwh$ *Cylinder:* $V = \pi r^2 h$ *Triangular Prism:* $V = Bh$ *Tetrahedron:* $V = ⅓\,Bh$	*Pyramid:* $V = ⅓\,Bh$ *Cone:* $V = ⅓\,bh = ⅓\,\pi r^2 h$ *Sphere:* $V = \frac{4}{3}\pi r^3$ *Ellipsoid:* $V = \frac{4}{3}\pi abc$	