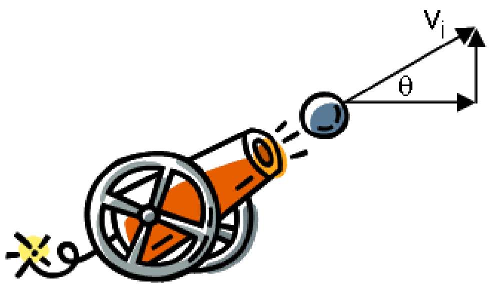
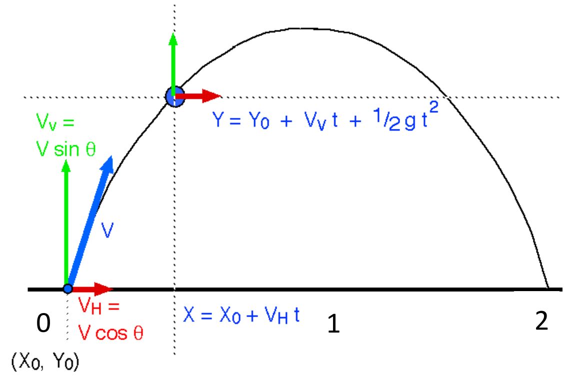
Harold's Physics of Projectiles
"Cheat Sheet"

26 November 2017

<table>
<tr><th colspan="3">The Classic Cannon Ball Problem</th></tr>
<tr><td>Diagram</td><td colspan="2">


</td></tr>
<tr><td>Givens</td><td colspan="2">$\boldsymbol{v} = 40\ {}^{m}/_{s}$
$\theta = 30^{\circ}$ Degrees inclined from horizontal</td></tr>
<tr><td rowspan="3">Unknowns</td><td>Horizontal (x-axis)</td><td>Vertical (y-axis)</td></tr>
<tr><td>❶ How far is it at time t? $(x(t))$
❹ How far will it land? (x_{max})</td><td>❷ How high is it at time t? $(y(t))$
❺ How high will it go? (y_{max})</td></tr>
<tr><td colspan="2">❸ When will it land? (t_{max})</td></tr>
<tr><td rowspan="3">Observations</td><td colspan="2">Notes:
Subscripts are dimensions, time, or both. Examples:
v_x is the velocity in the x direction.
x_0 is the initial horizontal position, or horizontal position at time = 0 s.
v_{y0} is the initial velocity in the y direction (vertical)
Horizontal and vertical dimensions are orthogonal (independent from one another).
Assume no wind resistance (drag). If we factored in wind resistance, then differential calculus is needed.
The cannon ball will reach its highest point exactly half way through its journey. [t_1 and x_1]</td></tr>
<tr><td>$x_0 = 0, \quad x_1 = \frac{1}{2}x_2, \quad x_2 = x_{max}$
$v_x = v_{x0} = v_{x1} = v_{x2} = constant$
$a_x = 0$</td><td>$y_0 = 0, \quad y_1 = y_{max}, \quad y_2 = 0$
$v_{y0} = ?, \quad v_{y1} = 0, \quad v_{y2} = -v_{y0}$
$a_y = \boldsymbol{g} = -9.8\ {}^{m}/_{s^2}$</td></tr>
<tr><td colspan="2">$t_0 = 0, \quad t_1 = \frac{1}{2}t_2, \quad t_2 = t_{max}$</td></tr>
<tr><td rowspan="2">Equations</td><td>$\boldsymbol{x} = x_0 + v_{x0}t + \frac{1}{2}\boldsymbol{a}_x t^2$</td><td>$\boldsymbol{y} = y_0 + v_{y0}t + \frac{1}{2}\boldsymbol{a}_y t^2$</td></tr>
<tr><td>$v_x = \boldsymbol{v}\cos(\theta)$</td><td>$v_y = \boldsymbol{v}\,sin(\theta)$</td></tr>
</table>

We are now ready to solve for all 5 unknowns in the order 1,2,3,4,5.

	Horizontal (x-axis)	Vertical (y-axis)
Solve	$\boldsymbol{x} = \cancel{x_0 +} v_{x0}t \cancel{+ \frac{1}{2}\boldsymbol{a}_x t^2}$ $\boldsymbol{x} = v_{x0}t$	$\boldsymbol{y} = \cancel{y_0 +} v_{y0}t - \frac{1}{2}gt^2$ $\boldsymbol{y} = v_{y0}t - \frac{1}{2}gt^2$
	$x(t) = v_{x0}t = \boldsymbol{v}\,cos(\theta)\,t$	$y(t) = \boldsymbol{v}\sin(\theta)\,t - \frac{1}{2}gt^2$
Substitute	$x(t) = 40\,cos(30^{\circ})\,t\ \ m$	$y(t) = 40\,sin(30^{\circ})\,t - 4.9\,t^2\,m$
Box Answer	❶ $\boxed{x(t) = 40\,cos(30^{\circ})\,t\ \ m}$ Distance travelled	❷ $\boxed{y(t) = 40\,sin(30^{\circ})\,t - 4.9\,t^2\,m}$ Height travelled

Solve		$\boldsymbol{y}(t_0) = y_0 = 0 = v_{y0}t - \frac{1}{2}gt^2$ $(t)\left(v_{y0} - \frac{1}{2}gt\right) = 0$ $t = t_0 = 0, \quad t = t_2 = \frac{2v_{y0}}{g}$ $t_{max} = t_2 = \frac{2v_{y0}}{g} = \frac{2(\boldsymbol{v}\,sin(\theta))}{g}$
Substitute	$t_{max} = \frac{2(40\,sin(30^{\circ}))}{9.8} = 4.08\,s$	
Box Answer	❸ $\boxed{t_{max} = 4.08\,s}$ Time the cannon ball was in the air	

Solve	$x_{max} = v_{x0}\,t_{max} = \boldsymbol{v}\,cos(\theta)\,t_{max}$	$y_{max} = y(t_1) = y\left(\frac{1}{2}t_{max}\right)$ $y_{max} = 40\,sin(30^{\circ})\left(\frac{1}{2}t_{max}\right) - 4.9\left(\frac{1}{2}t_{max}\right)^2$
Substitute	$x_{max} = 40\,cos(30^{\circ})\,4.08 = 141.3\,m$	$y_{max} = 40\sin(30^{\circ})\left(\frac{4.08}{2}\right) - 4.9\left(\frac{4.08}{2}\right)^2$ $= 20.41\,m$
Box Answer	❹ $\boxed{x_{max} = 141.3\,m}$ Farthest distance the cannon ball travelled	❺ $\boxed{y_{max} = 20.41\,m}$ Highest distance the cannon ball travelled