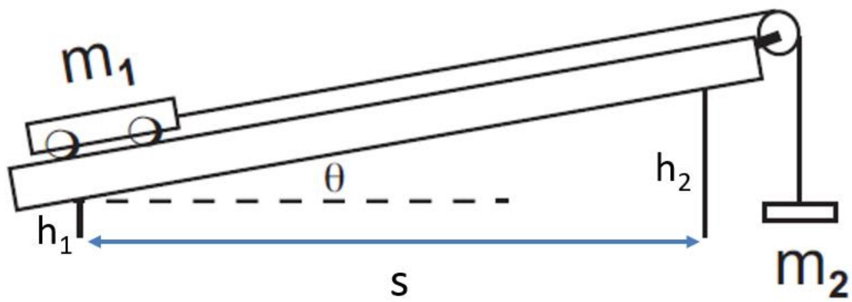
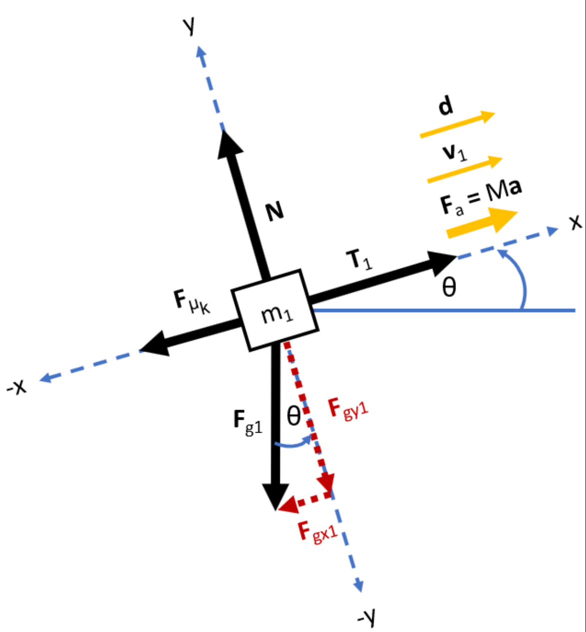
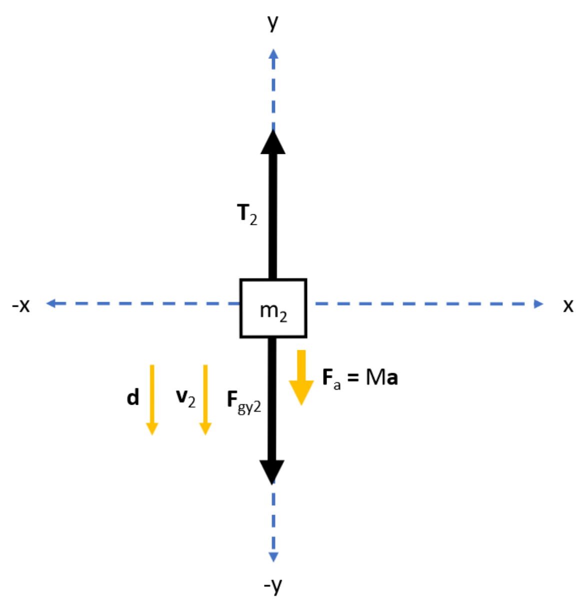
Harold's Physics of Forces with Pully
"Cheat Sheet"

23 April 2021

Incline Plane with a Pully		
Physical Setup		
Free Body Diagrams		
Givens	$m_1 = 560\ g = 0.560\ kg$ $\theta = 13.5^{\circ}$ $\mu_k = 0.003895$ $\mu_{pully} = 0$ $m_{string} = 0\ kg$ $h_1, h_2, and\ s = ramp\ leg\ heights\ and\ length$	$m_2 = 140\ g = 0.140\ kg$ $x_0 = 16.2\ cm = 0.162\ m$ $x = 78.7\ cm = 0.787\ m$ $\boldsymbol{v}_0 = 0.127\frac{cm}{s} = 0.00127\ \frac{m}{s}$ $\boldsymbol{v} = 145\frac{cm}{s} = 1.45\ \frac{m}{s}$
Observations	$\boldsymbol{g} = 9.80\ \frac{m}{s^2}$ $\theta = \tan^{-1}\left(\frac{h_2 - h_1}{s}\right)$	$\lvert\boldsymbol{d}_1\rvert = \lvert\boldsymbol{d}_2\rvert$ $\lvert\boldsymbol{v}_1\rvert = \lvert\boldsymbol{v}_2\rvert$ $\lvert\boldsymbol{T}_1\rvert = \lvert\boldsymbol{T}_2\rvert$ $\boldsymbol{T}_1 - \boldsymbol{T}_2 = 0$
Unknowns	$\boldsymbol{T}_1 = ?\ N$ $W_{net} = ?\ J$ $\Delta K = ?\ J$	$\boldsymbol{T}_2 = ?\ N$ $\boldsymbol{F}_a = ?\ N$ $\boldsymbol{a} = ?\ \frac{m}{s^2}$

<table>
<tr><td rowspan="3">Equations</td><td colspan="2">$\sum \boldsymbol{F} = \boldsymbol{F}_a = m\boldsymbol{a}$</td></tr>
<tr><td>$\sum \boldsymbol{F}_{x1} = \boldsymbol{T}_1 - \boldsymbol{F}_{gx1} - \boldsymbol{F}_{\mu_k} = \boldsymbol{F}_a$

$\sum \boldsymbol{F}_{y1} = \boldsymbol{N} - \boldsymbol{F}_{gy1} = 0$</td><td>$\sum \boldsymbol{F}_{x2} = 0$

$\sum \boldsymbol{F}_{y2} = \boldsymbol{T}_2 - \boldsymbol{F}_{gy2} = \boldsymbol{F}_a$</td></tr>
<tr><td>$\boldsymbol{F}_{g1} = m_1\,\boldsymbol{g}$
$\boldsymbol{F}_{gx1} = \boldsymbol{F}_{g1}\,\sin\theta$
$\boldsymbol{F}_{gy1} = \boldsymbol{F}_{g1}\,\cos\theta$
$\boldsymbol{F}_{\mu_k} = \mu_k\,\boldsymbol{N}$

$W_{net} = W_{\mu 1} + W_{g1} + W_{g2} + W_{T1} + W_{T2}$
$W = \boldsymbol{Fd}\cos\theta$

$\Delta K = K_f - K_0$
$K = \frac{1}{2}mv^2$

$W_{net} = \Delta K$</td><td>$\boldsymbol{F}_{gy2} = m_2\,\boldsymbol{g}$

$\boldsymbol{F}_a = M\boldsymbol{a}$
$M = m_1 + m_2$

$d = x - x_0$</td></tr>
<tr><td>Solve

1) Forces</td><td>$\boldsymbol{F}_{gx1} = m_1\,\boldsymbol{g}\sin\theta$
$\boldsymbol{F}_{gy1} = m_1\,\boldsymbol{g}\cos\theta$

$\sum \boldsymbol{F}_{y1} = \boldsymbol{N} - \boldsymbol{F}_{gy1} = 0$
$\boldsymbol{N} = \boldsymbol{F}_{gy1}$
$\boldsymbol{N} = m_1\,\boldsymbol{g}\cos\theta$

$\boldsymbol{F}_{\mu_k} = \mu_k\,\boldsymbol{N}$
$\boldsymbol{F}_{\mu_k} = \mu_k\,m_1\,\boldsymbol{g}\cos\theta$

$\sum \boldsymbol{F}_{x1} = \boldsymbol{T}_1 - \boldsymbol{F}_{gx1} - \boldsymbol{F}_{\mu_k} = \boldsymbol{F}_a$
$\boldsymbol{F}_a = \boldsymbol{T}_1 - \boldsymbol{F}_{gx1} - \boldsymbol{F}_{\mu_k}$
$\boldsymbol{F}_a = \boldsymbol{T}_1 - m_1\,\boldsymbol{g}\sin\theta - \mu_k\,m_1\,\boldsymbol{g}\cos\theta$</td><td>$\sum \boldsymbol{F}_{y2} = \boldsymbol{T}_2 - \boldsymbol{F}_{gy2} = -\boldsymbol{F}_a$
$\boldsymbol{T}_2 - \boldsymbol{F}_{gy2} = -\boldsymbol{F}_a$
$-\boldsymbol{T}_2 + m_2\,\boldsymbol{g} = \boldsymbol{F}_a$
$\boldsymbol{F}_a = m_2\,\boldsymbol{g} - \boldsymbol{T}_2$

$\boldsymbol{T}_1 - \boldsymbol{T}_2 = 0$
$\boldsymbol{T} = \lvert\boldsymbol{T}_1\rvert = \lvert\boldsymbol{T}_2\rvert$

$\boldsymbol{F}_a = M\boldsymbol{a}$
$\boldsymbol{F}_a = (m_1 + m_2)\,\boldsymbol{a}$
$\boldsymbol{a} = \frac{\boldsymbol{F}_a}{m_1 + m_2}$

$\boldsymbol{d} = x - x_0 = 0.787\,m - 0.162\,m$
$\boldsymbol{d} = 0.625\,m$</td></tr>
<tr><td>2) Tension</td><td colspan="2">$\boldsymbol{T} - m_1\,\boldsymbol{g}\sin\theta - \mu_k\,m_1\,\boldsymbol{g}\cos\theta = \boldsymbol{F}_a$
$m_2\,\boldsymbol{g} - \boldsymbol{T} = \boldsymbol{F}_a$

$\boldsymbol{T} - m_1\,\boldsymbol{g}\sin\theta - \mu_k\,m_1\,\boldsymbol{g}\cos\theta = m_2\,\boldsymbol{g} - \boldsymbol{T}$
$2\,\boldsymbol{T} = m_2\,\boldsymbol{g} + m_1\,\boldsymbol{g}\sin\theta + \mu_k\,m_1\,\boldsymbol{g}\cos\theta$
$2\,\boldsymbol{T} = \boldsymbol{g}\,[m_2 + m_1\sin\theta + \mu_k\,m_1\cos\theta]$
$\boldsymbol{T} = \left(\frac{\boldsymbol{g}}{2}\right)[m_2 + m_1\sin\theta + \mu_k\,m_1\cos\theta]$
$\boldsymbol{T} = \lvert\boldsymbol{T}_1\rvert = \lvert\boldsymbol{T}_2\rvert$</td></tr>
</table>

3) Work-Energy	$$W_{\mu 1} = \boldsymbol{F}_{\mu_k} \boldsymbol{d} \cos 180^\circ = -\boldsymbol{F}_{\mu_k} \boldsymbol{d}$$ $$W_{g1} = \boldsymbol{F}_{gx1} \boldsymbol{d} \cos 180^\circ = -\boldsymbol{F}_{gx1} \boldsymbol{d}$$ $$W_{g2} = \boldsymbol{F}_{gy2} \boldsymbol{d} \cos 0^\circ = \boldsymbol{F}_{gy2} \boldsymbol{d}$$ $$W_{T1} = \boldsymbol{T}_1 \boldsymbol{d} \cos 0^\circ = \boldsymbol{T}_1 \boldsymbol{d}$$ $$W_{T2} = \boldsymbol{T}_2 \boldsymbol{d} \cos 180^\circ = -\boldsymbol{T}_2 \boldsymbol{d}$$ $$W_{net} = W_{\mu 1} + W_{g1} + W_{g2} + W_{T1} + W_{T2}$$ $$W_{net} = -\boldsymbol{F}_{\mu_k} \boldsymbol{d} - \boldsymbol{F}_{gx1} \boldsymbol{d} + \boldsymbol{F}_{gy2} \boldsymbol{d} + \boldsymbol{T}_1 \boldsymbol{d} - \boldsymbol{T}_2 \boldsymbol{d}$$ $$W_{net} = \boldsymbol{d} \left[-\boldsymbol{F}_{\mu_k} - \boldsymbol{F}_{gx1} + \boldsymbol{F}_{gy2} + \boldsymbol{T}_1 - \boldsymbol{T}_2\right]$$ $$\boldsymbol{T}_1 - \boldsymbol{T}_2 = 0$$ $$W_{net} = \boldsymbol{d} \left[\boldsymbol{F}_{gy2} - \boldsymbol{F}_{\mu_k} - \boldsymbol{F}_{gx1}\right]$$ $$W_{net} = \boldsymbol{d} \left[m_2 \boldsymbol{g} - \mu_k m_1 \boldsymbol{g} \cos\theta - m_1 \boldsymbol{g} \sin\theta\right]$$ $$W_{net} = \boldsymbol{d} \boldsymbol{g} \left[m_2 - \mu_k m_1 \cos\theta - m_1 \sin\theta\right]$$ $$\Delta K = K_f - K_0$$ $$\Delta K = \frac{1}{2} m \boldsymbol{v}_f^2 - \frac{1}{2} m \boldsymbol{v}_0^2$$ $$\Delta K = \frac{1}{2} m \left(\boldsymbol{v}_f^2 - \boldsymbol{v}_0^2\right)$$
Substitute	$$\boldsymbol{T} = \left(\frac{\boldsymbol{g}}{2}\right) \left[m_2 + m_1 \sin\theta + \mu_k m_1 \cos\theta\right] = \boldsymbol{T}_1 = \boldsymbol{T}_2$$ $$\boldsymbol{T} = \left(\frac{9.8 \frac{m}{s^2}}{2}\right) \left[(0.140\ kg) + (0.560\ kg) \sin 13.5^\circ + (0.003895)(0.560\ kg) \cos 13.5^\circ\right]$$ $$\boldsymbol{T} = \boldsymbol{T}_1 = \boldsymbol{T}_2 = \boldsymbol{1.337\ N}$$
	$$\boldsymbol{F}_a = m_2 \boldsymbol{g} - \boldsymbol{T}_2$$ $$\boldsymbol{F}_a = (0.140\ kg)\left(9.80\ {}^{m}/_{s^2}\right) - 1.337\ N$$ $$\boldsymbol{F}_a = \boldsymbol{0.0350\ N}$$ *or* $$\boldsymbol{F}_a = \boldsymbol{T} - m_1 \boldsymbol{g} \sin\theta - \mu_k m_1 \boldsymbol{g} \cos\theta$$ $$\boldsymbol{F}_a = 1.358\ N - (0.560\ kg)\left(9.80 \frac{m}{s^2}\right) \sin 13.5^\circ - (0.003895)(0.560\ kg)\left(9.80 \frac{m}{s^2}\right) \cos 13.5^\circ$$ $$\boldsymbol{F}_a = \boldsymbol{0.0350\ N}$$
	$$\boldsymbol{a} = \frac{\boldsymbol{F}_a}{m_1 + m_2}$$ $$\boldsymbol{a} = \frac{0.0350\ N}{(0.560\ kg) + (0.140\ kg)}$$ $$\boldsymbol{a} = \boldsymbol{0.0500 \frac{m}{s^2}}$$

<table>
<tr><td rowspan="2">Substitute</td><td colspan="2">$$W_{net} = \boldsymbol{d}\,\boldsymbol{g}\,[m_2 - \mu_k\, m_1 \cos\theta - m_1 \sin\theta]$$
$$W_{net} = (0.625\,m)\left(9.8\frac{m}{s^2}\right)\left[0.140\,kg - (0.003895)(0.560\,kg)\cos 13.5^{\circ} - (0.560\,kg)\sin 13.5^{\circ}\right]$$
$$\boldsymbol{W_{net} = 0.0438\,J}$$</td></tr>
<tr><td colspan="2">$$\Delta K = \frac{1}{2}M\left(\boldsymbol{v}_f^2 - \boldsymbol{v}_0^2\right)$$
$$\Delta K = \frac{1}{2}(m_1 + m_2)\left(\boldsymbol{v}_f^2 - \boldsymbol{v}_0^2\right)$$
$$\Delta K = \frac{1}{2}(0.560\,kg + 0.140\,kg)\left(\left(1.45\frac{m}{s}\right)^2 - \left(0.00127\frac{m}{s}\right)^2\right)$$
$$\boldsymbol{\Delta K = 0.7359\,J}$$</td></tr>
<tr><td>Box Your Answers</td><td>$\boxed{\boldsymbol{T}_1 = 1.337\,N}$
$\boxed{W_{net} = 0.0438\,J}$
$\boxed{\Delta K = 0.7359\,J}$</td><td>$\boxed{\boldsymbol{T}_2 = 1.337\,N}$
$\boxed{\boldsymbol{F}_a = 0.0350\,N}$
$\boxed{\boldsymbol{a} = 0.0500\,\frac{m}{s^2}}$</td></tr>
<tr><td>Check Your Answers</td><td colspan="2">https://amesweb.info/Physics/Inclined-Plane-Calculator.aspx</td></tr>
<tr><td>Analysis</td><td>$Ideally, W_{net} = \Delta K$
$W_{net} = Theoretical = 0.0438\,J$
$\Delta K = Experimental = 0.7359\,J$

$$\%\,Error = \left|\frac{Theoretical - Experimental}{Theoretical}\right|$$
$$\%\,Error = \left|\frac{W_{net} - \Delta K}{W_{net}}\right|(100\%)$$
$$\%\,Error = \left|\frac{0.0438\,J\; - 0.7359\,J}{0.0438\,J}\right|(100\%)$$
$$\boldsymbol{\%\,Error = 1{,}580\,\%}$$
• *Measurements are off by a factor of ~17.*
• *Theoretical value grossly under estimates expirimental value.*
• *Probably due mostly to pully rotational energy not being added to* W_{net}.</td><td>**Possible sources of error:**
1. The pully really does have friction. [The pully friction pulls Work (-W) out of the system in the form of heat energy]
2. The pully spins and has rotational energy. [$K = \frac{1}{2}I\omega^2$ *should be added to* W_{net}]
3. The string really does have mass. [$W = \boldsymbol{F}\,\boldsymbol{d} = \left(m_1 + m_2 + m_{string}\right)\boldsymbol{g}\,\boldsymbol{d}$]
4. Accumulating string mass contributes to vertical W_{Fgy}. [$\boldsymbol{F}_{gy2} = \left(m_2 + m_{string}\right)\boldsymbol{g}$]
5. Gravity (**g**) varies by altitude and geo location. [9.8 vs. 9.81]
6. Measuring equipment margin of error. [±0.0001]
7. Human error in measurements. [±0.01]
8. Air resistance as the masses move.</td></tr>
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