

Harold's Partial Fraction Decomposition (Precalculus)

Cheat Sheet

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Partial Fractions	http://en.wikipedia.org/wiki/Partial_fraction_decomposition
Condition	$f(x) = \frac{P(x)}{Q(x)} = \frac{ax^n + \dots + b}{cx^m + \dots + d}$ where $P(x)$ and $Q(x)$ are polynomials
Preparation	Case 1: $n \geq m$, Perform long division first Case 2: $n < m$, Proceed to the cases below
Case I: Simple linear (1 st degree)	$\frac{A}{(ax + b)} \quad \text{e.g.,} \quad \frac{A}{x}, \quad \frac{A}{x + 1}$
Case II: Multiple degree linear (1 st degree)	$\frac{A}{(ax + b)} + \frac{B}{(ax + b)^2} + \frac{C}{(ax + b)^3}$
Case III: Simple quadratic (2 nd degree)	$\frac{Ax + B}{(ax^2 + bx + c)} \quad \text{e.g.,} \quad \frac{3x}{(x^2 + 9)}$
Case IV: Multiple degree quadratic (2 nd degree)	$\frac{Ax + B}{(ax^2 + bx + c)} + \frac{Cx + D}{(ax^2 + bx + c)^2} + \frac{Ex + F}{(ax^2 + bx + c)^3}$
Example Expansion	$\frac{P(x)}{(ax + b)(cx + d)^2(ex^2 + fx + g)}$ $= \frac{A}{(ax + b)} + \frac{B}{(cx + d)} + \frac{C}{(cx + d)^2} + \frac{Dx + E}{(ex^2 + fx + g)}$

Steps to Solve	Precalculus Example
1. Write down problem	$\frac{5x + 1}{2x^2 - x - 1}$
2. Check if long division is needed	Not needed since degree of numerator (top) is less than degree of denominator (bottom)
3. Factor the denominator	$\frac{5x + 1}{(2x + 1)(x - 1)}$
4. Expand with A, B, Cs	$\frac{5x + 1}{(2x + 1)(x - 1)} = \frac{A}{(2x + 1)} + \frac{B}{(x - 1)}$
5. Find a common denominator	$= \frac{A(x - 1)}{(2x + 1)(x - 1)} + \frac{B(2x + 1)}{(2x + 1)(x - 1)}$
6. Focus on numerator	$5x + 1 = A(x - 1) + B(2x + 1)$
7. FOIL if necessary	$(x + 1)(x - 2) = x^2 - x - 2$

8. Expand/distribute the A, B, Cs	$5x + 1 = Ax - A + 2Bx + B$
9. Regroup by powers of x. (x^2, x, c)	$5x + 1 = Ax + 2Bx - A + B$
10. Factor by powers of x. ($\quad)x^2 + (\quad)x + (c)$	$(5)x + (1) = (A + 2B)x + (-A + B)$
11. Introduce ghost factors if needed (0, 1)	$5x + 1 = (0)x^2 + (5)x + (1)$
12. Match left and right coefficients for a system of equations	$A + 2B = 5$ $-A + B = 1$
13. Solve system of equations	Pick simplest method below
a. Substitution method	$B = A + 1$ $A + 2(A + 1) = 5$ $A + 2A + 2 = 5$ $3A = 3$ $A = 1$ $B = A + 1 = 1 + 1 = 2$ A = 1 B = 2
b. Row elimination method	$A + 2B = 5$ $+ [-A + B = 1]$ <hr/> $3B = 6$ B = 2 $A + 2B = 5$ $-2 [-A + B = 1]$ <hr/> $3A = 3$ A = 1
c. Augmented matrix method	$\left[\begin{array}{cc c} A & B & k \\ A & B & k \end{array} \right] = \left[\begin{array}{cc c} 1 & 2 & 5 \\ -1 & 1 & 1 \end{array} \right]$ Use TI-84 <i>rref()</i> function $= \left[\begin{array}{cc c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array} \right]$ A = 1 B = 2
14. Reassemble newly expanded function with values for A, B, C	$\frac{5x + 1}{(2x + 1)(x - 1)} = \frac{1}{(2x + 1)} + \frac{2}{(x - 1)}$
15. Verify function for accuracy	Verify the two equations are the same by plugging in any value for x and see if f(x) is the same for both.