Harold’s Matrix Cheat Sheet

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Matrix Definitions

Property	Example
Dimension	3 columns $A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix}$ 2 rows $Dim = rows \ \times columns = 2 \times 3$
Vector	$n \times 1$ Matrix
Zero Matrix	$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Identity Matrix (I_n)	$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Matrix Elements	
Rank	
Row Matrix	A Matrix in which there is only one row and no column.
Column Matrix	A Matrix in which there is only one column and no row.
Horizontal Matrix	A Matrix in which the number of rows is less than the number of columns.
Vertical Matrix	A Matrix in which the number of columns is less than the number of rows.
Rectangular Matrix	A Matrix in which the number of rows and columns are unequal.
Square Matrix	A matrix in which the number of rows and columns are the same.
Diagonal Matrix	A square matrix in which the non-diagonal elements are zero.
Zero or Null Matrix	A matrix whose all elements are zero.
Unit or Identity Matrix	A diagonal matrix whose all diagonal elements are 1.
Symmetric Matrix	A square matrix where the transpose of the original matrix is equal to its original matrix. i.e. $(A^T) = A$.
Skew-symmetric Matrix	A skew-symmetric (or antisymmetric or antimetric[1]) matrix is a square matrix whose transpose equals its negative i.e. $(A^T) = -A$.

Orthogonal Matrix	$AA^T = A^TA = I_n$
Idempotent Matrix	$A^2 = A$
Involutory Matrix	$A^2 = I_n$
Upper Triangular Matrix	A square matrix in which all the elements below the diagonal are zero.
Lower Triangular Matrix	A square matrix in which all the elements above the diagonal are zero.
Singular Matrix	A square matrix whose determinant is zero. i.e. $\|A\| = 0$
Nonsingular Matrix	A square matrix whose determinant is non-zero. i.e. $\|A\| \neq 0$

Matrix Properties

Property	Example
Matrix Addition	
Commutative	$A + B = B + A$
Associative	$(A + B) + C = A + (B + C)$
Additive Identity	For any matrix A, there is a unique matrix O such that $A + 0 = A$.
Additive Inverse	For each A, there is a unique matrix $-A$ such that $A + (-A) = 0$.
Closure	$A + B$ is a matrix of the same dimensions as A and B.
Scalar Multiplication	
Associative	$(cd)A = c(dA)$ $c(AB) = (cA)B = A(cB)$
Distributive	$c(A + B) = cA + cB$ $(c + d)A = cA + dA$
Multiplicative Identity	$1A = A$
Multiplicative Properties of Zero	$0 \cdot A = O$ $c \cdot O = O$
Closure	cA is a matrix of the same dimensions as A.
Matrix Multiplication	
Not Commutative	$AB \neq BA$
Associative	$(AB)C = A(BC)$
Distributive	$A(B + C) = AB + AC$ $(B + C)A = BA + CA$
Multiplicative Identity	$I_nA = A$ $AI_n = A$
Multiplicative Property of Zero	$OA = O$ $AO = O$
Dimension	The product of an $m \times n$ matrix and an $n \times k$ matrix is an $m \times k$ matrix.
Transpose	
Inverse	$(A^T)^T = A$

Addition	$(A+B)^T = A^T + B^T$
Constant Multiple	$(cA)^T = cA^T$
Multiplication	$(AB)^T = B^T A^T$ (Note reverse order)
Identity	${I_n}^T = I_n$
Inverse (Square Matrix)	
Inverse	$(A^{-1})^{-1} = A$ $AA^{-1} = A^{-1}A = I_n$
Distributuve	$(cA)^{-1} = c^{-1}A^{-1}, \qquad r \neq 0$
Multiplication	$(AB)^{-1} = B^{-1}A^{-1}$ (Note reverse order, A and B must be invertable)
Identity	${I_n}^{-1} = I_n$
Commutative	$(A^T)^{-1} = (A^{-1})^T$
Adjoint (Square Matrix)	
	$A(Adj\ (A)) = (Adj\ (A))A = \lvert A\rvert I_n$
	$Adj(AB) = (Adj\ (B)) \cdot (Adj\ (A))$
	$\lvert Adj\ (A)\rvert = \lvert A\rvert^{n-1}$
	$Adj(kA) = k^{n-1}Adj(A)$
	$\left\lvert Adj\left(Adj(A)\right)\right\rvert = \lvert A\rvert(n-1)^2$
	$Adj(Adj(A)) = A^{(n-2)} \times A$
	$If\ A = [l, m, n] then\ Adj(A) = [MN, LN, LM]$
	$Adj(I_n) = I_n$

Matrix Operations

Property	Example
Augmented Matrix	
Transpose	
Determinant	
Dot Product	
Cross Product	
Adjoint	
Norm	
Eigen Values and Eigen Vectors	

Sources:

Matrices: Definition, Properties, Types, Formulas, and Examples (geeksforgeeks.org)