Harold’s Fundamental Theorem of Calculus
Cheat Sheet
23 April 2024

The First Fundamental Theorem of Calculus: Integrating Derivatives

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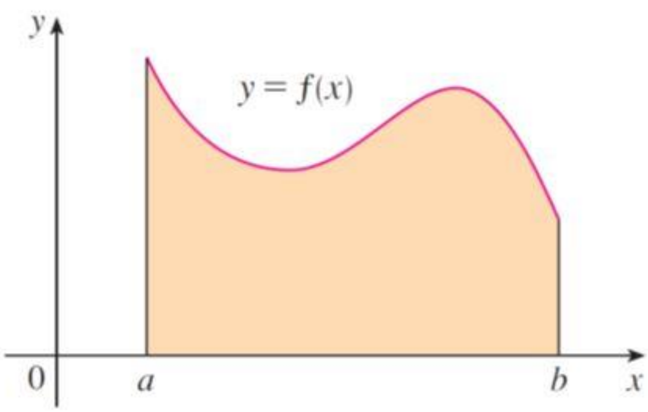
Upper Bound Minus Lower Bound Formula

$$\int_a^b \boldsymbol{f(x)\,dx} = \int_a^b \boldsymbol{F'(x)\,dx} = \boldsymbol{F(x)|_a^b} = \boldsymbol{F(b) - F(a)}$$



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Solve:

$$\int_0^2 x^2 + 1\,dx$$

1st FToC Formula

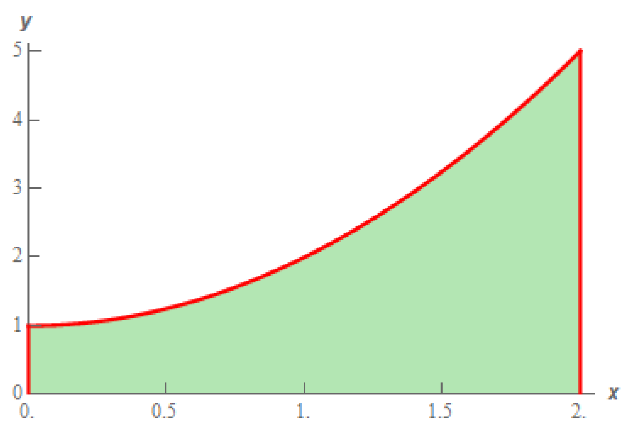
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Identify functions

$$f(x) = F'(x) = x^2 + 1$$
$$F(x) = \frac{x^3}{3} + x$$

Plug into formula

$$F(b) - F(a)$$
$$= F(2) - F(0)$$
$$= \left[\frac{(2)^3}{3} + (2)\right] - \left[\frac{(0)^3}{3} + 0\right]$$
$$= \frac{14}{3} - 0 = \frac{14}{3} \approx 4.6667$$



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The Second Fundamental Theorem of Calculus: Differentiating Integrals

Formula	Example
Simple Formula $$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$ Single Function Formula $$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f\big(g(x)\big)g'(x)$$ General Formula $$\boldsymbol{\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)}$$	**Solve:** $$\frac{d}{dx}\int_{4x}^{x^2} e^t\,dt$$ 2^{nd} FToC General Formula $$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f\big(h(x)\big)\,h'(x) - f\big(g(x)\big)\,g'(x)$$ Identify functions $$f(t) = e^t$$ $$g(x) = 4x$$ $$h(x) = x^2$$ Substitute $$f\big(g(x)\big) = e^{4x}$$ $$f\big(h(x)\big) = e^{x^2}$$ Differentiate $$g'(x) = \frac{d}{dx}4x = 4$$ $$h'(x) = \frac{d}{dx}x^2 = 2x$$ Plug into formula $$= f\big(h(x)\big)\,h'(x) - f\big(g(x)\big)\,g'(x)$$ $$= e^{x^2}(2x) - e^{4x}(4)$$ $$= \boldsymbol{2xe^{x^2} - 4e^{4x}}$$
Proof: a) Apply the First Fundamental Theorem of Calculus $$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = \frac{d}{dx}[F(t)]_{g(x)}^{h(x)}$$ $$= \frac{d}{dx}\Big[F\big(h(x)\big) - F\big(g(x)\big)\Big]$$ b) Distribute $$= \frac{d}{dx}F\big(h(x)\big) - \frac{d}{dx}F\big(g(x)\big)$$ c) Chain rule $$= F'\big(h(x)\big)\,\frac{d}{dx}h(x) - F'\big(g(x)\big)\,\frac{d}{dx}g(x)$$ d) Simplify $$= F'\big(h(x)\big)\,h'(x) - F'\big(g(x)\big)\,g'(x)$$ e) Substitute $$= f\big(h(x)\big)\,h'(x) - f\big(g(x)\big)\,g'(x)$$	