Harold's Finances
"Cheat Sheet"
29 March 2022

	Variable Descriptions
Variables	$PV = Original\ amount, principle, or\ present\ value$ $FV = Amount\ after\ time\ t, future\ value, or\ face\ value$ $r = Annual\ interest\ rate, rate\ of\ growth\ or\ loss\ (15\% = 0.15)$ $k = Number\ of\ periods\ or\ times\ per\ year\ (quarterly\ is\ k = 4)$ $t = Number\ of\ years$ $n = Number\ of\ periods\ or\ compoundings\ (n = kt)$ $i = Effective\ interest\ rate\ per\ period\ \left(i = \frac{r}{k}\right)$ $x = Number\ of\ payment\ already\ made$ $e = Euler's\ number\ (\sim 2.71828\ 18284\ 59045\ \ldots)$ $PMT = R = Equal\ regular\ payments\ towards\ a\ loan\ or$ $Equal\ periodic\ payments\ from\ an\ annuity$ $BAL = B = Remaining\ balance\ on\ a\ loan\ or\ annuity$ $DIS = Discount\ on\ a\ U.S. Treasury\ bill\ (T - bill)$ $APY = Annual\ Percentage\ Yield\ (APY)\ or\ Effective\ Interest\ Rate$

One-Time Investment	Formulas	
Simple Interest	**Discrete**	**Continuous**
Simple Interest	$I = PVrt$	NA
Future Value	$FV = PV + I$ $FV = PV + PVrt$ $FV = PV(1 + rt)$	
Present Value	$PV = \frac{FV}{(1 + rt)}$ $PV = FV(1 + rt)^{-1}$	
T-Bill	$DIS = FVrt$ $Price = FV - DIS = FV(1 - rt)$ $Effective\ Rate = \frac{DIS}{PVt} \bullet 100\ \%$	

One-Time Investment	Formulas	
Compounded Interest	**Discrete**	**Continuous**
Compounded Interest	$I = FV - PV$ $I = PV((1 + r)^t - 1)$	$I = PV(e^{rt} - 1)$
Future Value	$FV = PV\left(1 + \frac{r}{k}\right)^{kt}$ *If k = 1 (annually) then* $FV = PV(1 + r)^t$	$FV = PVe^{rt}$

Present Value	$PV = \frac{FV}{\left(1+\left(\frac{r}{k}\right)\right)^{kt}} = PV\left(1+\left(\frac{r}{k}\right)\right)^{-kt}$ *If k = 1 (annually) then* $PV = \frac{FV}{(1+r)^t} = FV\ (1+r)^{-t}$	$PV = \frac{FV}{e^{rt}}$ $PV = FVe^{-rt}$
Annual Interest Rate	$r = k\left[\left(\frac{FV}{PV}\right)^{-kt} - 1\right] \bullet 100\ \%$	$r = \frac{1}{t} ln\left(\frac{FV}{PV}\right)$
Annual Percentage Yield (APY) or Effective Interest Rate	$\%\ APY = \left[\left(1+\left(\frac{r}{k}\right)\right)^{kt} - 1\right] \bullet 100\ \%$ $APY = r_E = (1+i)^k - 1$	$i = ln\left(\frac{FV}{PV}\right)$

Regular Payments	Formulas	
Compounded Interest	**Future Value**	**Present Value**
Number of Periods or Compoundings	$n = kt$	
Effective Interest Rate Per Period	$i = \frac{r}{k}$	
Cost of Loan (Amount You Paid)	$Total_{Paid} = ktPMT$	
Interest You Paid	$I_{Paid} = ktPMT - PV$	
Value of an Ordinary Annuity (PMT at end of period)	$FV = PMT\left[\frac{\left(\left(1+\left(\frac{r}{k}\right)\right)^{kt} - 1\right)}{\left(\frac{r}{k}\right)}\right]$ $FV = PMT\left[\frac{((1+i)^n - 1)}{i}\right]$	$PV = PMT\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt}\right)}{\left(\frac{r}{k}\right)}\right]$ $PV = PMT\left[\frac{(1-(1+i)^{-n})}{i}\right]$
Value of an Annuity Due (PMT at beginning of period)	$FV = PMT\left[\frac{\left(\left(1+\left(\frac{r}{k}\right)\right)^{kt+1} - 1\right)}{\left(\frac{r}{k}\right)}\right] - PMT$ $FV = PMT\left[\frac{((1+i)^{n+1} - 1)}{i}\right] - PMT$	$PV = PMT + PMT\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt+1}\right)}{\left(\frac{r}{k}\right)}\right]$ $PV = PMT + PMT\left[\frac{(1-(1+i)^{-(n-1)})}{i}\right]$
Amortization Payment Amount	$PMT = FV\left[\frac{\left(\left(1+\left(\frac{r}{k}\right)\right)^{kt} - 1\right)}{\left(\frac{r}{k}\right)}\right]^{-1}$ $PMT = FV\left[\frac{i}{((1+i)^n - 1)}\right]$	$PMT = PV\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt}\right)}{\left(\frac{r}{k}\right)}\right]^{-1}$ $PMT = PV\left[\frac{i}{(1-(1+i)^{-n})}\right]$
Remaining Balance		$BAL = PMT\left[\frac{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt+x}\right)}{\left(\frac{r}{k}\right)}\right]$ $BAL = PMT\left[\frac{(1-(1+i)^{-(n-x)})}{i}\right]$

Examples	Calculations
Savings Account: $PV = \$100$ $r = 8\% = 0.08$ $k = 4\ (quarterly)$ $t = 1\ year$	$If\ k = 1,\ FV = \$108.00\ (+0¢)\ Annually$ $If\ k = 4,\ FV = \$108.24\ (+24¢)\ Quarterly$ $If\ k = 12,\ FV = \$108.30\ (+6¢)\ Monthly$ $If\ k = 52,\ FV = \$108.32\ (+2¢)\ Weekly$ $If\ k = 365,\ FV = \$108.33\ (+1¢)\ Daily$ $If\ k \to \infty,\ FV = \$108.33\ (+0¢)\ Continuously$
House Mortgage Payment: $PV = \$300{,}000\ (home\ loan)$ $PMT = Equal\ periodic\ payments$ $r = 3.5\% = 0.035$ $k = 12\ (monthly)$ $t = 30\ years$	$PMT = PV\left[\frac{\left(\frac{r}{k}\right)}{\left(1-\left(1+\left(\frac{r}{k}\right)\right)^{-kt}\right)}\right]$ $PMT = \$300{,}000\left[\frac{\left(\frac{0.035}{12}\right)}{\left(1-\left(1+\left(\frac{0.035}{12}\right)\right)^{-(12)(30)}\right)}\right]$ $\boldsymbol{PMT = \$1,347.13/month}$
Loan Cost Analysis	$\underline{t = 30\ years}$: $Cost\ of\ loan = ktPMT = (12)(30)(\$1{,}347.13) = \$484{,}966.80$ $Interest\ paid = ktPMT - PV = \$484{,}966.80 - \$300{,}000 = \$184{,}966.80$ $\underline{t = 15\ years}$: $Cost\ of\ loan = ktPMT = (12)(15)(\$2{,}144.65) = \$386{,}037.00$ $Interest\ paid = ktPMT - PV = \$386{,}037.00 - \$300{,}000 = \$86{,}037.00$

