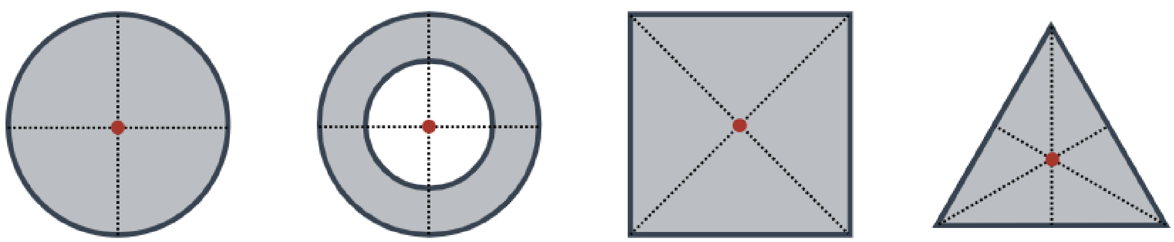
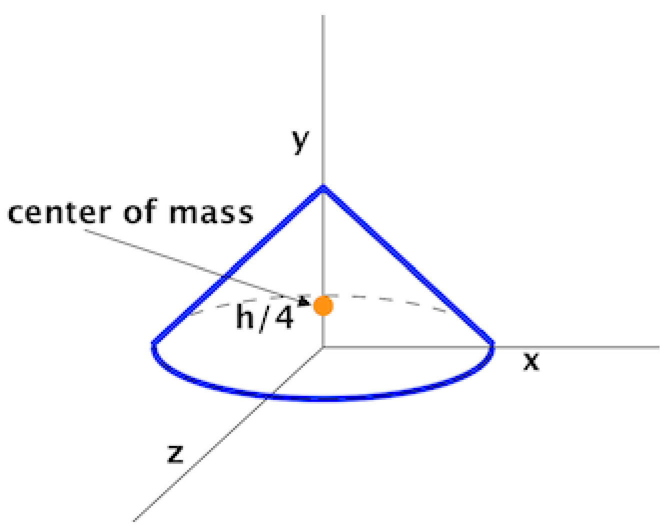
Harold's Center of Mass
Cheat Sheet
13 August 2020

Terms

Term	Description	Units
$\bar{x}$	Center of Mass along the x-axis.	$m \text{ or } ft$
$\bar{y}$	Center of Mass along the y-axis.	$m \text{ or } ft$
$\bar{z}$	Center of Mass along the z-axis.	$m \text{ or } ft$
$\bar{x}$ $(\bar{x}, \bar{y})$ $(\bar{x}, \bar{y}, \bar{z})$	Center of Mass. The coordinates (point) where the object is perfectly balanced. (Centroid)	$m \text{ or } ft$
M	Mass. How heavy the object is. Is equal to it's area or volume if uniform density. (similar to Weight)	$kg \text{ or } lb$
$M_x \text{ and } M_y$	Moment. The line or axis on which the object can spin perfectly balanced.	$kg \text{ or } lb$
M_x	Moment of Inertia about the x-axis	$kg \text{ or } lb$
M_y	Moment of Inertia about the y-axis	$kg \text{ or } lb$
I	Moment of Inertia, mass moment of Inertia, or rotational inertia of a body	$kg\, m^2 \text{ or } lb\, ft^2$
ρ	Greek symbol rho for density or mass / volume.	$\rho = \frac{kg}{m^3} \text{ or } \frac{lb}{ft^3}$





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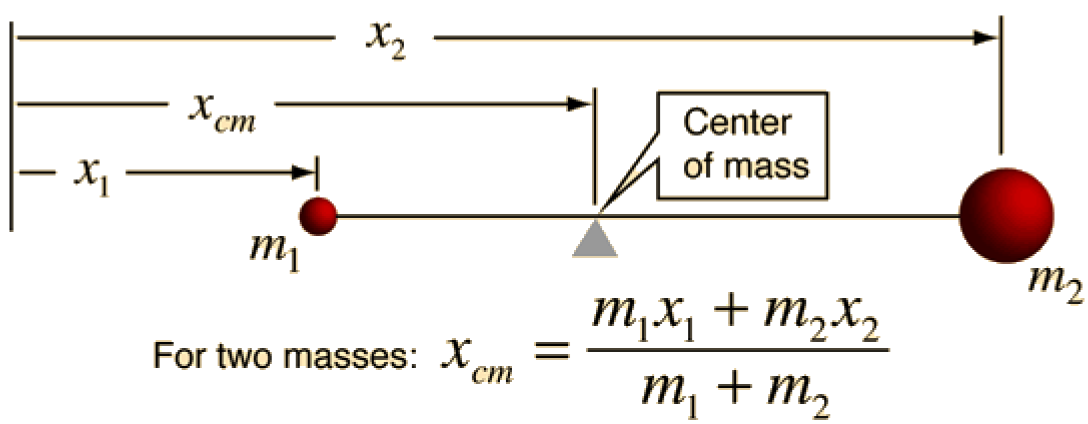
Discrete

Term	1-D	2-D	3-D
M	$M = \sum_{i=1}^{N} m_i = total\ mass$		
M_x	$M_x = \sum_{i=1}^{N} m_i\, y_i$		
M_y	$M_y = \sum_{i=1}^{N} m_i\, x_i$		
$\bar{x}$ x_{cm}	$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$	$\bar{x} = \frac{M_y}{M}$	$\bar{x} = \frac{1}{M} \sum_{i=1}^{N} m_i\, x_i$
$\bar{y}$	NA	$\bar{y} = \frac{M_x}{M}$	$\bar{y} = \frac{1}{M} \sum_{i=1}^{N} m_i\, y_i$
$\bar{z}$	NA	NA	$\bar{z} = \frac{1}{M} \sum_{i=1}^{N} m_i\, z_i$
$\boldsymbol{R}$	NA	$\boldsymbol{R} = \frac{1}{M} \sum_{i=1}^{N} m_i\, r_i$	NA
I	$I = m\, r^2 = \frac{L}{\omega}$	$I = \sum_{i=1}^{N} m_i\, r_i^2$	NA



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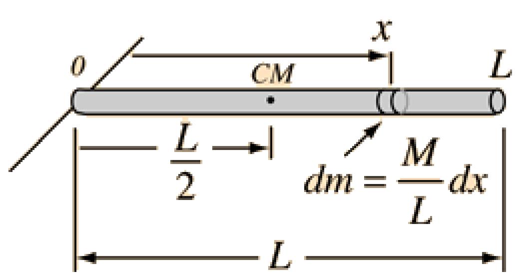
Continuous

Term	2-D	3-D
M	$M = \rho\ (Area\ of\ plate\ or\ laminate)$ $M = \rho \int_a^b f(x)\, dx$	$M = \int_0^M dm$ $dm = \rho\, dz\, dy\, dx$
M_x	$M_x = \rho \int_a^b \frac{1}{2}([f(x)]^2)\ dx$	NA
M_y	$M_y = \rho \int_a^b x\ f(x)\ dx$	NA
$\bar{x}$	$\bar{x} = \frac{M_y}{M}$	$\bar{x} = \frac{1}{M}\int_0^M x\, dm$
$\bar{y}$	$\bar{y} = \frac{M_x}{M}$	$\bar{y} = \frac{1}{M}\int_0^M y\, dm$
$\bar{z}$	NA	$\bar{z} = \frac{1}{M}\int_0^M z\, dm$ $\bar{z} = \frac{1}{V}\int_{z_{min}}^{z_{max}} z\, dV$
$\boldsymbol{R}$	$\boldsymbol{R} = \frac{1}{M}\int \boldsymbol{r}\, dm$ $\boldsymbol{R} = \frac{1}{M}\iint_A \rho(\boldsymbol{r})\, \boldsymbol{r}\, dA$ *Where* $\boldsymbol{r}$ *is distance from the axis of rotation, not origin.*	$\boldsymbol{R} = \frac{1}{M}\int \boldsymbol{r}\, dm$ $\boldsymbol{R} = \frac{1}{M}\iiint_V \rho(\boldsymbol{r})\, \boldsymbol{r}\, dV$ *Where* $\boldsymbol{r}$ *is distance from the axis of rotation, not origin.*
I	$I = \int_0^a m\, r^2\, dr$ $I = \iint_A \rho(\boldsymbol{r})\, d(\boldsymbol{r})^2\, dA(\boldsymbol{r})$	$I = \iiint_V \rho(\boldsymbol{r})\, d(\boldsymbol{r})^2\, dV(\boldsymbol{r})$

$$x_{cm} = \frac{\int_0^L x \frac{M}{L} dx}{M} = \frac{1}{L}\frac{x^2}{2}\Bigg|_{x=0}^{x=L} = \frac{L}{2}$$



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