

Harold's AP Calculus Notes

Cheat Sheet

23 April 2024

Limits	
<p>Definition of Limit Let f be a function defined on an open interval containing c and let L be a real number. The statement:</p> $\lim_{x \rightarrow c} f(x) = L$ <p>means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that</p> $\text{if } x - c < \delta, \text{ then } f(x) - L < \epsilon$ <p>Tip: Direct substitution: Plug in $f(c)$ and see if it provides a legal answer. If so then $L = f(c)$.</p>	
<p>The Existence of a Limit The limit of $f(x)$ as x approaches c is L if and only if:</p>	$\lim_{x \rightarrow c^-} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c^+} f(x) = L$
<p>Definition of Continuity A function f is continuous at c if for every $\epsilon > 0$ there exists a $\delta > 0$ such that $x - c < \delta$ and $f(x) - f(c) < \epsilon$.</p> <p>Tip: Rearrange $f(x) - f(c)$ to have $x - c$ as a factor. Since $x - c < \delta$ we can find an equation that relates both δ and ϵ together.</p>	<p>Prove that $f(x) = x^2 - 1$ is a continuous function.</p> $\begin{aligned} & f(x) - f(c) \\ &= (x^2 - 1) - (c^2 - 1) \\ &= x^2 - 1 - c^2 + 1 \\ &= x^2 - c^2 \\ &= (x + c)(x - c) \\ &= x + c x - c \\ &= x + c x - c \end{aligned}$ <p>Since $x + c \leq 2c$ $f(x) - f(c) \leq 2c x - c < \epsilon$</p> <p>So, given $\epsilon > 0$, we can choose $\delta = \left \frac{1}{2c} \right \epsilon > 0$ in the Definition of Continuity. So, substituting the chosen δ for $x - c$ we get:</p> $ f(x) - f(c) \leq 2c \left(\left \frac{1}{2c} \right \epsilon \right) = \epsilon$ <p>Since both conditions are met, $f(x)$ is continuous.</p>
<p>Two Special Trig Limits</p>	$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

Derivatives	(See Larson's 1-pager of common derivatives)
Definitions of a Derivative of a Function (Slope Function)	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$
Derivatives Notation	$\frac{dy}{dx}, y', f'(x), f^{(n)}(x), \frac{d}{dx}[f(x)], D_x[y]$
1. Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$
2. Constant Rule	$\frac{d}{dx}[c] = 0$
3. Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = cf'(x)$
4. Sum and Difference Rule	$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
5. Product Rule	$\frac{d}{dx}[fg] = fg' + gf'$
6. Quotient Rule	$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$
7. Power Rule	$\frac{d}{dx}[x^n] = nx^{n-1}$
8. General Power Rule	$\frac{d}{dx}[u^n] = nu^{n-1} u' \text{ where } u = u(x)$
9. Power Rule for x	$\frac{d}{dx}[x] = 1 \text{ (think } x = x^1 \text{ and } x^0 = 1)$
10. Absolute Value	$\frac{d}{dx}[x] = \frac{x}{ x }$
11. Natural Exponential Rule	$\frac{d}{dx}[e^x] = e^x$
12. General Natural Exponential Rule	$\frac{d}{dx}[e^{g(x)}] = e^{g(x)} g'(x)$
13. Exponential Rule	$\frac{d}{dx}[a^x] = (\ln a) a^x$
14. General Exponential Rule	$\frac{d}{dx}[a^{g(x)}] = (\ln a) a^{g(x)} g'(x)$
15. Natural Logarithm Rule	$\frac{d}{dx}[\ln x] = \frac{1}{x}$
16. General Natural Logarithm Rule	$\frac{d}{dx}[\ln f(x)] = \frac{1}{f(x)} \cdot f'(x)$
17. Logarithm Rule	$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a) x}$
18. General Logarithm Rule	$\frac{d}{dx}[\log_a f(x)] = \frac{1}{\ln a} \cdot \frac{f'(x)}{f(x)}$

19. Sine	$\frac{d}{dx}[\sin(x)] = \cos(x)$
20. Cosine	$\frac{d}{dx}[\cos(x)] = -\sin(x)$
21. Tangent	$\frac{d}{dx}[\tan(x)] = \sec^2(x)$
22. Cotangent	$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$
23. Secant	$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$
24. Cosecant	$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$
25. Arcsine	$\frac{d}{dx}[\sin^{-1}(x)] = \frac{1}{\sqrt{1-x^2}}$
26. Arccosine	$\frac{d}{dx}[\cos^{-1}(x)] = \frac{-1}{\sqrt{1-x^2}}$
27. Arctangent	$\frac{d}{dx}[\tan^{-1}(x)] = \frac{1}{1+x^2}$
28. Arccotangent	$\frac{d}{dx}[\cot^{-1}(x)] = \frac{-1}{1+x^2}$
29. Arcsecant	$\frac{d}{dx}[\sec^{-1}(x)] = \frac{1}{ x \sqrt{x^2-1}}$
30. Arccosecant	$\frac{d}{dx}[\csc^{-1}(x)] = \frac{-1}{ x \sqrt{x^2-1}}$
31. Hyperbolic Sine $\left(\frac{e^x - e^{-x}}{2}\right)$	$\frac{d}{dx}[\sinh(x)] = \cosh(x)$
32. Hyperbolic Cosine $\left(\frac{e^x + e^{-x}}{2}\right)$	$\frac{d}{dx}[\cosh(x)] = \sinh(x)$
33. Hyperbolic Tangent	$\frac{d}{dx}[\tanh(x)] = \operatorname{sech}^2(x)$
34. Hyperbolic Cotangent	$\frac{d}{dx}[\coth(x)] = -\operatorname{csch}^2(x)$
35. Hyperbolic Secant	$\frac{d}{dx}[\operatorname{sech}(x)] = -\operatorname{sech}(x)\tanh(x)$
36. Hyperbolic Cosecant	$\frac{d}{dx}[\operatorname{csch}(x)] = -\operatorname{csch}(x)\coth(x)$
37. Hyperbolic Arcsine	$\frac{d}{dx}[\sinh^{-1}(x)] = \frac{1}{\sqrt{x^2+1}}$
38. Hyperbolic Arccosine	$\frac{d}{dx}[\cosh^{-1}(x)] = \frac{1}{\sqrt{x^2-1}}$
39. Hyperbolic Arctangent	$\frac{d}{dx}[\tanh^{-1}(x)] = \frac{1}{1-x^2}$
40. Hyperbolic Arccotangent	$\frac{d}{dx}[\coth^{-1}(x)] = \frac{1}{1-x^2}$
41. Hyperbolic Arcsecant	$\frac{d}{dx}[\operatorname{sech}^{-1}(x)] = \frac{-1}{x\sqrt{1-x^2}}$

42. Hyperbolic Arccosecant	$\frac{d}{dx} [\operatorname{csch}^{-1}(x)] = \frac{-1}{ x \sqrt{1+x^2}}$
----------------------------	--

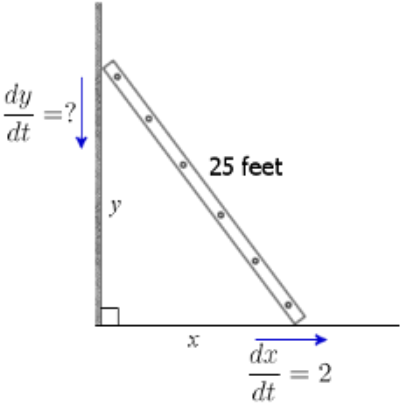

Physics	Translational Motion
Position Function	$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$ $x(t) = \frac{1}{2}a_x t^2 + v_{0,x}t + x_0$ $y(t) = \frac{1}{2}a_y t^2 + v_{0,y}t + y_0$
Velocity Function	$v(t) = s'(t) = gt + v_0$
Acceleration Function	$a(t) = v'(t) = s''(t)$
Jerk Function	$j(t) = a'(t) = v''(t) = s^{(3)}(t)$
Gravitational Constant (g)	$g \approx -9.81 \frac{m}{s^2}$ $g \approx -32.2 \frac{ft}{s^2}$

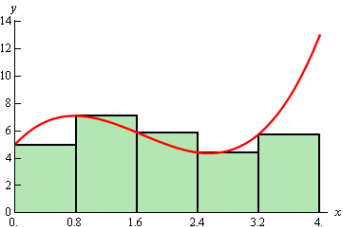
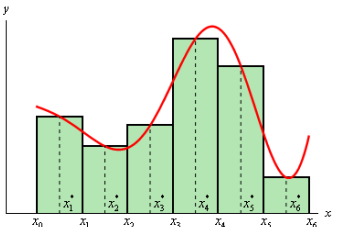
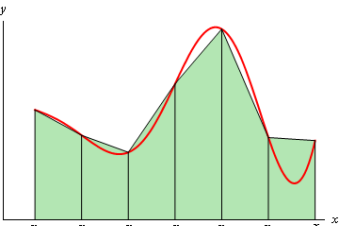
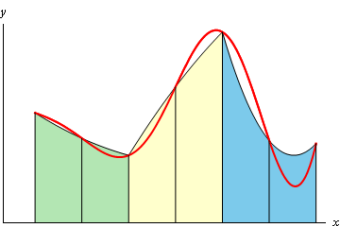


Analyzing the Graph of a Function	(See Harold's Illegals and Graphing Rationals Cheat Sheet)
x-Intercepts (Zeros or Roots)	$f(x) = 0$
y-Intercept	$f(0) = y$
Domain	Valid x values
Range	Valid y values
Continuity	No division by 0, no negative square roots or logs
Vertical Asymptotes (VA)	$x =$ division by 0 or undefined
Horizontal Asymptotes (HA)	$\lim_{x \rightarrow \infty^-} f(x) \rightarrow y$ and $\lim_{x \rightarrow \infty^+} f(x) \rightarrow y$
Infinite Limits at Infinity	$\lim_{x \rightarrow \infty^-} f(x) \rightarrow \infty$ and $\lim_{x \rightarrow \infty^+} f(x) \rightarrow \infty$
Differentiability	Limit from both directions arrives at the same slope
Relative Extrema	Create a table with domains: $f(x), f'(x), f''(x)$
Concavity	If $f''(x) \rightarrow +$, then cup up \cup If $f''(x) \rightarrow -$, then cup down \cap
Points of Inflection	$f''(x) = 0$, then concavity changes

Graphing with Derivatives	
Test for Increasing and Decreasing Functions	<ol style="list-style-type: none"> If $f'(x) > 0$, then f is increasing (slope up) \nearrow If $f'(x) < 0$, then f is decreasing (slope down) \searrow If $f'(x) = 0$, then f is constant (zero slope) \rightarrow
First Derivative Test	<ol style="list-style-type: none"> If $f'(x)$ changes from $-$ to $+$ at c, then f has a relative minimum at $(c, f(c))$ If $f'(x)$ changes from $+$ to $-$ at c, then f has a relative maximum at $(c, f(c))$ If $f'(x)$ is $+$ at c or $-$ at c, then $f(c)$ is neither
Second Derivative Test Let $f'(c) = 0$, and $f''(x)$ exists, then	<ol style="list-style-type: none"> If $f''(x) > 0$, then f has a relative minimum at $(c, f(c))$ If $f''(x) < 0$, then f has a relative maximum at $(c, f(c))$ If $f''(x) = 0$, then the test fails
Test for Concavity	<ol style="list-style-type: none"> If $f''(x) > 0$ for all x, then the graph is concave up \cup If $f''(x) < 0$ for all x, then the graph is concave down \cap
Inflection Points (Change in concavity)	If $(c, f(c))$ is a point of inflection of $f(x)$, then either <ol style="list-style-type: none"> $f''(c) = 0$ or $f''(x)$ does not exist at $x = c$

Tangent Lines	
General Form	$ax + by + c = 0$
Slope-Intercept Form	$y = mx + b$
Point-Slope Form	$y - y_0 = m(x - x_0)$ where $m = f'(x_0)$ at point (x_0, y_0)
Calculus Form	$f(x) = f'(c)(x - c) + f(c)$
Slope	$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \rightarrow \frac{dy}{dx} = f'(x)$

Differentiation & Differentials		
Rolle's Theorem	<p>Assume f is <u>continuous</u> on the closed interval $[a, b]$, and f is <u>differentiable</u> on the open interval (a, b).</p> <p>If $f(a) = f(b)$, then there exists at least one number c in (a, b) such that $f'(c) = 0$.</p>	
Mean Value Theorem	<p>If f meets the conditions of Rolle's Theorem, then you can find 'c'.</p> $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{\Delta y}{\Delta x}$ $f(b) = f(a) + (b - a)f'(c)$	
Intermediate Value Theorem	<p>Assume f is a <u>continuous</u> function with the interval $[a, b]$ as its domain.</p> <p>If f takes values $f(a)$ and $f(b)$ at each end of the interval, then it also takes any value between $f(a)$ and $f(b)$ at some point within the interval.</p>	
Calculating Differentials (Tangent line approximation)	$f(x + \Delta x) \approx f(x) + \Delta y$ <p>since $dy = f'(x) dx$ then $\Delta y \approx f'(x) \Delta x$ so</p> $f(x + \Delta x) \approx f(x) + f'(x) \Delta x$ $\text{Relative Error} = \frac{\Delta f}{f} \text{ in \%}$ <p>Example: $\sqrt[4]{82}$ $f(x) = \sqrt[4]{x}$ $f(x + \Delta x) = f(81 + 1)$</p>	
Newton's Method	<p>Finds zeros of f, or finds c if $f(c) = 0$.</p> $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ <p>Example: $\sqrt[4]{82}$ $f(x) = x^4 - 82 = 0$ start with $x_0 = 3$</p>	

<p>Related Rates</p>	<p>Steps to solve:</p> <ol style="list-style-type: none"> Identify the known variables and rates of change. $x = 15 \text{ m}$ $y = 20 \text{ m}$ $x' = 2 \frac{\text{m}}{\text{s}}$ $y' = \text{---} \frac{\text{m}}{\text{s}}$ Construct an equation relating these quantities. $x^2 + y^2 = r^2$ (Often uses the Pythagorean Theorem.) Differentiate both sides of the equation. $2xx' + 2yy' = 0$ Solve for the desired rate of change. $y' = -\frac{x}{y} x'$ Substitute the known rates of change and quantities into the equation. $y' = -\frac{15}{20} \cdot 2 = 1.5 \frac{\text{m}}{\text{s}}$ 	
<p>L'Hôpital's Rule</p>	<p>If $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} \frac{P(x)}{Q(x)}$ $= \left\{ \frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^\infty, 0^0, \infty^0, \infty - \infty \right\}$, but not $\{0^\infty\}$, then</p> $\lim_{x \rightarrow c} \frac{P(x)}{Q(x)} = \lim_{x \rightarrow c} \frac{P'(x)}{Q'(x)} = \lim_{x \rightarrow c} \frac{P''(x)}{Q''(x)} = \dots$	

Numerical Methods	
<p>Riemann Sum</p> 	$P_0(x) = \int_a^b f(x) dx = \lim_{\ P\ \rightarrow 0} \sum_{i=1}^n f(x_i^*) \Delta x_i$ <p>where $a = x_0 < x_1 < x_2 < \dots < x_n = b$ and $\Delta x_i = x_i - x_{i-1}$ and $\ P\ = \max\{\Delta x_i\}$</p> <p>Types:</p> <ul style="list-style-type: none"> • Left Sum (LHS) • Middle Sum (MHS) • Right Sum (RHS)
<p>Midpoint Rule (Middle Sum)</p> 	$P_0(x) = \int_a^b f(x) dx \approx \sum_{i=1}^n f(\bar{x}_i) \Delta x =$ $\Delta x [f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + \dots + f(\bar{x}_n)]$ <p>where $\Delta x = \frac{b-a}{n}$ and $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$</p> <p>Error Bounds: $E_M \leq \frac{K(b-a)^3}{24n^2}$</p>
<p>Trapezoidal Rule</p> 	$P_1(x) = \int_a^b f(x) dx \approx$ $\frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_3) + \dots + 2f(x_{n-1})$ $+ f(x_n)]$ <p>where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$</p> <p>Error Bounds: $E_T \leq \frac{K(b-a)^3}{12n^2}$</p>
<p>Simpson's Rule</p> 	$P_2(x) = \int_a^b f(x) dx \approx$ $\frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots$ $+ 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$ <p>Where n is even and $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$</p> <p>Error Bounds: $E_S \leq \frac{K(b-a)^5}{180n^4}$</p>
<p>TI-84 Plus</p> 	<p>[MATH] fnInt(f(x),x,a,b), [MATH] [1] [ENTER]</p> <p>Example: [MATH] fnInt(x^2,x,0,1)</p> $\int_0^1 x^2 dx = \frac{1}{3}$
<p>TI-Nspire CAS</p> 	<p>[MENU] [4] Calculus [3] Integral [TAB] [TAB] [X] [^] [2] [TAB] [TAB] [X] [ENTER]</p> <p>Shortcut: [ALPHA] [WINDOWS] [4]</p>

Integration	(See Harold's Fundamental Theorem of Calculus Cheat Sheet)
Basic Integration Rules (Integration is the "inverse" of differentiation, and vice versa.)	$\int f'(x) dx = f(x) + C$ $\frac{d}{dx} \int f(x) dx = f(x)$
$f(x) = 0$	$\int 0 dx = C$
$f(x) = k = kx^0$	$\int k dx = kx + C$
1. The Constant Multiple Rule	$\int k f(x) dx = k \int f(x) dx$
2. The Sum and Difference Rule	$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$
The Power Rule $f(x) = P_n(x) = kx^n + \dots$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ where } n \neq -1$ $\text{If } n = -1, \text{ then } \int x^{-1} dx = \ln x + C$
The General Power Rule	<p>If $u = g(x)$, and $u' = \frac{d}{dx} g(x)$ then</p> $\int u^n u' dx = \frac{u^{n+1}}{n+1} + C, \text{ where } n \neq -1$
Reimann Sum	$\sum_{i=1}^n f(c_i) \Delta x_i, \quad \text{where } x_{i-1} \leq c_i \leq x_i$
Definition of a Definite Integral (Area under curve)	$\lim_{\ \Delta\ \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) dx$ <p>where $\ \Delta\ = \Delta x = \frac{b-a}{n}$</p>
Swap Bounds	$\int_a^b f(x) dx = - \int_b^a f(x) dx$
Additive Interval Property	$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$
First Fundamental Theorem of Calculus	$\int_a^b f(x) dx = F(b) - F(a)$
Second Fundamental Theorem of Calculus	$\frac{d}{dx} \int_a^x f(t) dt = f(x)$ $\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x))g'(x)$ $\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt = f(h(x))h'(x) - f(g(x))g'(x)$
Mean Value Theorem for Integrals	$\int_a^b f(x) dx = f(c)(b-a) \quad \text{Find 'c'}$
Average Value of a Function	$\frac{1}{b-a} \int_a^b f(x) dx$

Integration Methods	
1. Memorized	See Ron Larson's 1-pager of common integrals
2. U-Substitution	$\int f(g(x))g'(x)dx = F(g(x)) + C$ Set $u = g(x)$, then $du = g'(x) dx$ $\int f(u) du = F(u) + C$ $u = \underline{\hspace{2cm}} \quad du = \underline{\hspace{2cm}} dx$
3. Integration by Parts	$\int u dv = uv - \int v du$ $u = \underline{\hspace{2cm}} \quad v = \underline{\hspace{2cm}}$ $du = \underline{\hspace{2cm}} \quad dv = \underline{\hspace{2cm}}$ <p>Pick 'u' using the LIATE Rule:</p> <p>L – Logarithmic : $\ln x, \log_b x$</p> <p>I – Inverse Trig.: $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x, \sec^{-1} x, etc.$</p> <p>A – Algebraic: $x^2, 3x^{60}, \sqrt{x}, etc.$</p> <p>T – Trigonometric: $\sin x, \cos x, \tan x, \sec x, \csc x, \cot x$</p> <p>E – Exponential: $e^x, 19^x$</p>
4. Partial Fractions	$\int \frac{P(x)}{Q(x)} dx$ where $P(x)$ and $Q(x)$ are polynomials. Case 1: If degree of $P(x) \geq Q(x)$ then do long division first. Case 2: If degree of $P(x) < Q(x)$ then do partial fraction expansion.
5a. Trig Substitution for $\sqrt{a^2 - x^2}$	$\int \sqrt{a^2 - x^2} dx$ Substitution: $x = a \sin \theta$ Identity: $1 - \sin^2 \theta = \cos^2 \theta$
5b. Trig Substitution for $\sqrt{x^2 - a^2}$	$\int \sqrt{x^2 - a^2} dx$ Substitution: $x = a \sec \theta$ Identity: $\sec^2 \theta - 1 = \tan^2 \theta$
5c. Trig Substitution for $\sqrt{x^2 + a^2}$	$\int \sqrt{x^2 + a^2} dx$ Substitution: $x = a \tan \theta$ Identity: $\tan^2 \theta + 1 = \sec^2 \theta$
6. Computer Algebra Systems (CAS)	TI-Nspire CX CAS Graphing Calculator TI-Nspire CAS iPad app
7. Numerical Methods	Riemann Sum, Midpoint Rule, Trapezoidal Rule, Simpson's Rule, various quadrature rules, TI-84 Calculator, etc.
8. WolframAlpha	WolframAlpha is the Google of mathematics. Shows steps. Free.
9. AI Chatbot	OpenAI ChatGPT , Microsoft Copilot , Google Gemini , etc.

Partial Fractions	(See Harold's Partial Fraction Decomposition Cheat Sheets)
Condition	$f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and degree of $P(x) < Q(x)$ If degree of $P(x) \geq Q(x)$ then do long division first
Example Expansion	$\frac{P(x)}{(ax+b)(cx+d)^2(ex^2+fx+g)}$ $= \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2} + \frac{Dx+E}{(ex^2+fx+g)}$
Typical Solution	$\int \frac{a}{x+b} dx = a \ln x+b + C$

Sequences & Series	(See Harold's Series Cheat Sheet)
Sequence	$\lim_{n \rightarrow \infty} a_n = L \text{ (Limit)}$ Example: $(a_n, a_{n+1}, a_{n+2}, \dots)$
Geometric Series	$S = \lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$ only if $ r < 1$ where r is the radius of convergence and $(-r, r)$ is the interval of convergence

Convergence Tests	(See Harold's Series Convergence Tests Cheat Sheet)										
Series Convergence Tests	<table border="0"> <tr> <td>1. Divergence or n^{th} Term</td> <td>6. Ratio</td> </tr> <tr> <td>2. Geometric Series</td> <td>7. Root</td> </tr> <tr> <td>3. p-Series</td> <td>8. Direct Comparison</td> </tr> <tr> <td>4. Alternating Series</td> <td>9. Limit Comparison</td> </tr> <tr> <td>5. Integral</td> <td>10. Telescoping Series</td> </tr> </table>	1. Divergence or n^{th} Term	6. Ratio	2. Geometric Series	7. Root	3. p-Series	8. Direct Comparison	4. Alternating Series	9. Limit Comparison	5. Integral	10. Telescoping Series
1. Divergence or n^{th} Term	6. Ratio										
2. Geometric Series	7. Root										
3. p-Series	8. Direct Comparison										
4. Alternating Series	9. Limit Comparison										
5. Integral	10. Telescoping Series										

Taylor Series	(See Harold's Taylor Series Cheat Sheet)
Taylor Series	$f(x) = P_n(x) + R_n(x)$ $= \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n + \frac{f^{(n+1)}(x^*)}{(n+1)!} (x-c)^{n+1}$ where $x \leq x^* \leq c$ (x^* is the worst case scenario or max. value of $f(x)$ in the range.) and $\lim_{x \rightarrow +\infty} R_n(x) = 0$