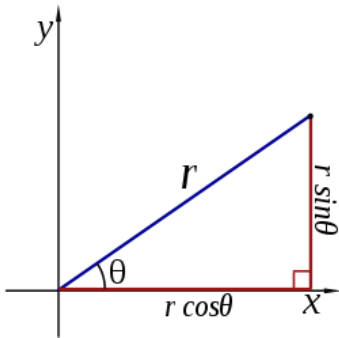


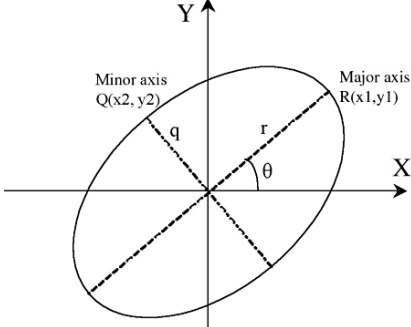
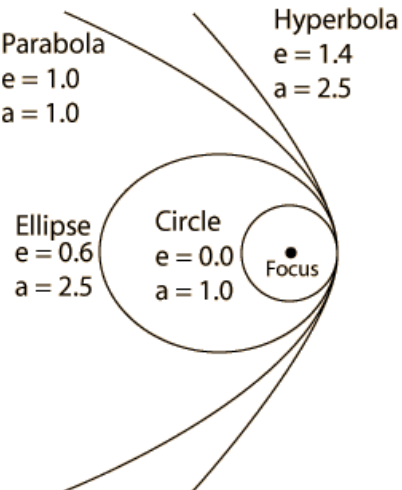
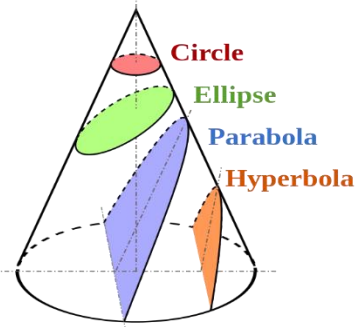
Harold's AP Calculus BC

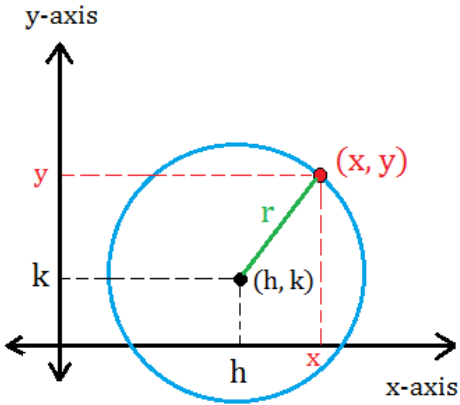
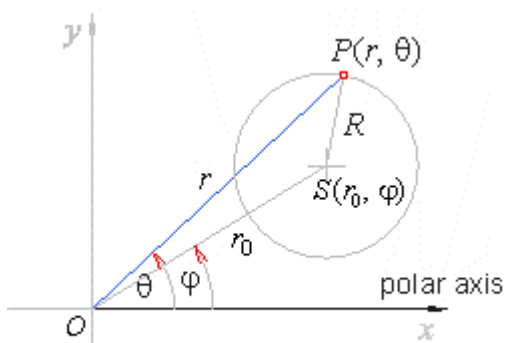
Cheat Sheet

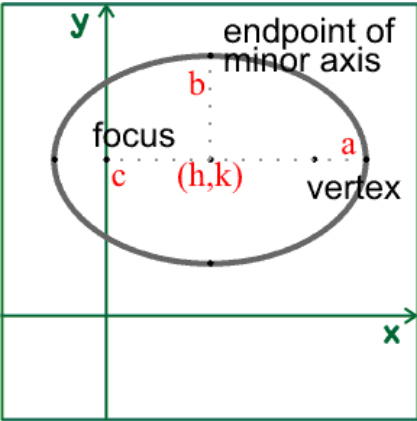
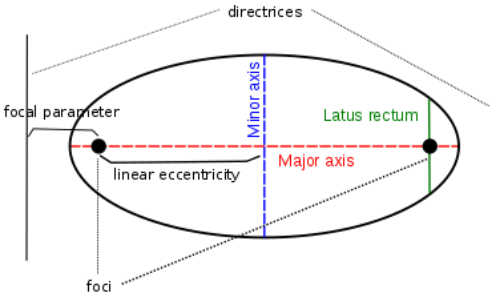
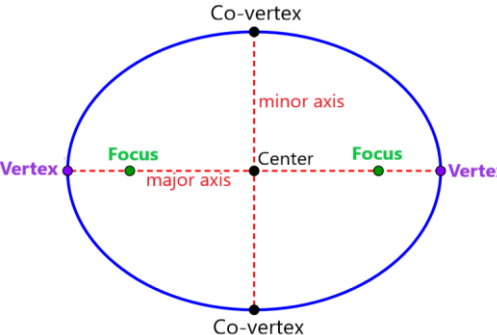
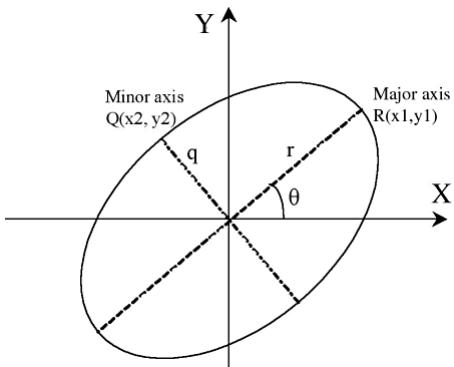
13 October 2025

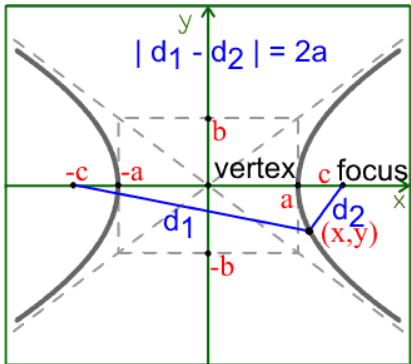
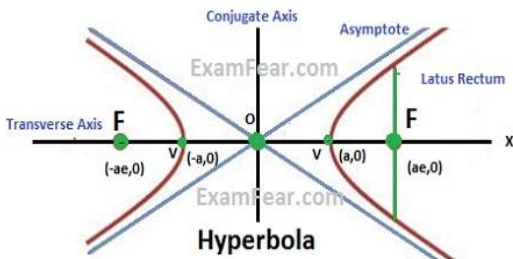
	Rectangular	Polar	Parametric
Point	$f(x) = y$ (x, y) (a, b) 	(r, θ) or $r \angle \theta$	
		$Polar \rightarrow Rect.$ $x = r \cos \theta$ $y = r \sin \theta$ $\tan \theta = \left(\frac{y}{x}\right)$	$Rect. \rightarrow Polar$ $r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1} \left(\frac{y}{x}\right)$
Line	<i>Slope-Intercept Form:</i> $y = mx + b$ <i>Point-Slope Form:</i> $y - y_0 = m(x - x_0)$ <i>Intercept Form:</i> $\frac{x}{a} + \frac{y}{b} = 1$ <i>Normal Form:</i> $Ax + By + C = 0$ <i>Calculus Form:</i> $f(x) = f'(a)x + f(0)$		
		<i>Point (a, b) in Rectangular:</i> $x(t) = a$ $y(t) = b$ $\langle a, b \rangle$ $t = 3^{rd}$, variable, usually time, with 1 degree of freedom (df) $\langle x, y \rangle = \langle x_0, y_0 \rangle + t \langle a, b \rangle$ $\langle x, y \rangle = \langle x_0 + at, y_0 + bt \rangle$ where $\langle a, b \rangle = \langle x_2 - x_1, y_2 - y_1 \rangle$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$	

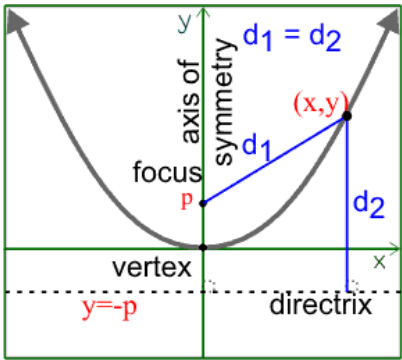
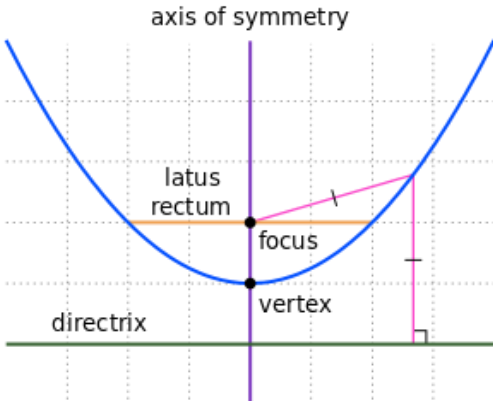
	Rectangular	Polar	Parametric
Plane	<i>Dot Product of Point-Normal Form:</i> $n_x(x - x_0) + n_y(y - y_0) + n_z(z - z_0) = 0$ where: $\mathbf{n} = \langle n_x, n_y, n_z \rangle = \langle a, b, c \rangle$ is orthogonal (perpendicular) to the plane <i>General Form:</i> $Ax + By + Cz + D = 0$ <i>Intercept Form:</i> $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	<i>Vector Form:</i> $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$	$\mathbf{r} = \mathbf{r}_0 + s\mathbf{v} + t\mathbf{w}$ <i>Parametric Form:</i> $x = x_0 + su_1 + tv_1$ $y = y_0 + su_2 + tv_2$ $z = z_0 + su_3 + tv_3$ where: • (x_0, y_0, z_0) is a point on the plane. • $\langle u_1, u_2, u_3 \rangle$ and $\langle v_1, v_2, v_3 \rangle$ are direction vectors on the plane. • s and t are parameters that vary over all real numbers.

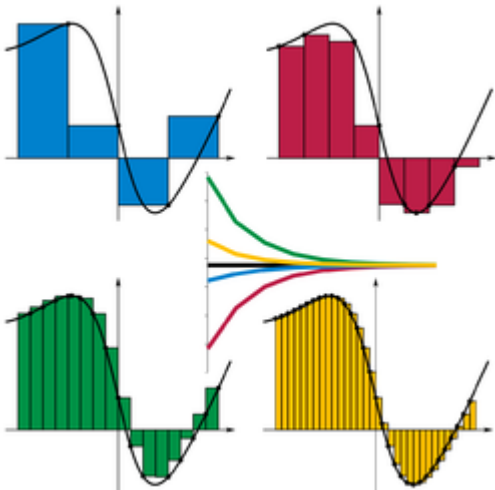
	Rectangular	Polar	Parametric
Conics	<i>General Equation for All Conics:</i> $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where Line: $A = B = C = 0$ Circle: $A = C$ and $B = 0$ Ellipse: $AC > 0$ or $B^2 - 4AC < 0$ Parabola: $AC = 0$ or $B^2 - 4AC = 0$ Hyperbola: $AC < 0$ or $B^2 - 4AC > 0$ Note: If $A + C = 0$, square hyperbola <i>Rotation:</i> If $B \neq 0$, then <u>rotate</u> the coordinate system: $\cot 2\theta = \frac{A - C}{B}$ $x = x' \cos \theta - y' \sin \theta$ $y = y' \cos \theta + x' \sin \theta$ New = (x', y'), Old = (x, y) rotates through angle θ from x-axis 	<i>General Equation for All Conics:</i> $r = \frac{p}{1 - e \cos \theta}$ where $p = \begin{cases} a(1 - e^2) \\ 2d \end{cases}$ for $\begin{cases} 0 \leq e < 1 \\ e = 1 \\ e > 1 \end{cases}$ p = semi-latus rectum or the line segment running from the focus to the curve in a direction parallel to the directrix <i>Eccentricity:</i> Circle $e = 0$ Ellipse $0 < e < 1$ Parabola $e = 1$ Hyperbola $e > 1$ 	 Circle Ellipse Parabola Hyperbola  

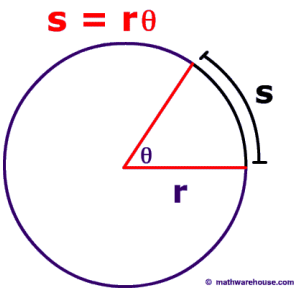
	Rectangular	Polar	Parametric
Circle	$x^2 + y^2 = r^2$ $(x - h)^2 + (y - k)^2 = r^2$ Center: (h, k) Vertices: NA Focus: (h, k) 	Centered at Origin: $r = a$ (constant) $\theta = \theta \quad [0, 2\pi] \text{ or } [0, 360^\circ]$ Centered at (r_0, ϕ): $r^2 + r_0^2 - 2rr_0 \cos(\theta - \phi) = R^2$ Hint: Law of Cosines or $r = r_0 \cos(\theta - \phi) + \sqrt{a^2 - r_0^2 \sin^2(\theta - \phi)}$ 	$x(t) = r \cos(t) + h$ $y(t) = r \sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi]$ Center: (h, k) Focus: (h, k)

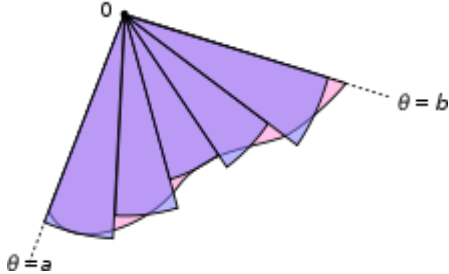
	Rectangular	Polar	Parametric
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Center: (h, k) Vertices: $(h \pm a, k)$ Co-Vertices: $(h, k \pm b)$ Foci: $(h \pm c, k)$ Focus length, c, from center: $c^2 = a^2 - b^2$ 	Ellipse: $r = \frac{a(1 - e^2)}{1 + e \cos \theta} \text{ for } 0 < e < 1$ where $e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$ relative to center (h, k)   Interesting Note: The <u>sum</u> of the distances from each focus to a point on the curve is constant. $d_1 + d_2 = k$	Center: (h, k) Rotated Ellipse: $x(t) = a \cos t \cos \theta - b \sin t \sin \theta + h$ $y(t) = a \cos t \sin \theta + b \sin t \cos \theta + k$ θ = the angle between the x-axis and the major axis of the ellipse 

	Rectangular	Polar	Parametric
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Center: (h, k) Vertices: $(h \pm a, k)$ Foci: $(h \pm c, k)$ Focus length, c, from center: $c^2 = a^2 + b^2$  Interesting Note: The <u>difference</u> between the distances from each focus to a point on the curve is constant. $d_1 - d_2 = k$	Vertical Axis of Symmetry: $r = \frac{a(e^2 - 1)}{1 + e \cos \theta}$ for $e > 1$ Eccentricity: $e > 1$ where $e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a} = \sec \theta > 1$ relative to center (h, k)  $p = \text{semi-latus rectum}$ or the line segment running from the focus to the curve in the directions $\theta = \pm \frac{\pi}{2}$	Left-Right Opening Hyperbola: $x(t) = a \sec(t) + h$ $y(t) = b \tan(t) + k$ $[t_{\min}, t_{\max}] = [-c, c]$ Vertex: (h, k) Up-Down Opening Hyperbola: $x(t) = a \tan(t) + h$ $y(t) = b \sec(t) + k$ $[t_{\min}, t_{\max}] = [-c, c]$ Vertex: (h, k) General Form: $x(t) = At^2 + Bt + C$ $y(t) = Dt^2 + Et + F$ where A and D have different signs

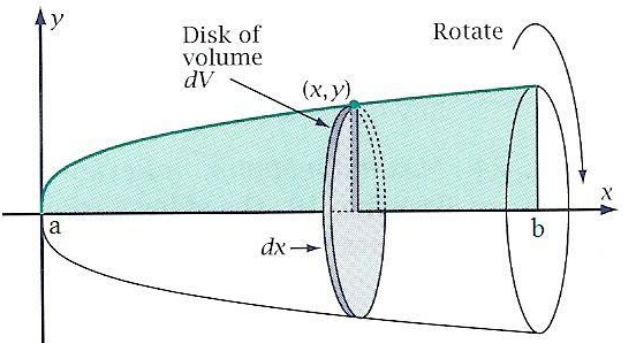
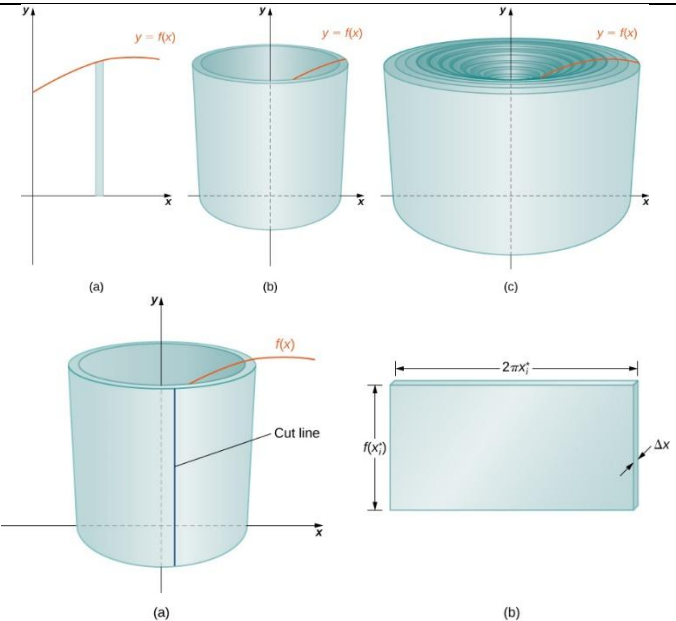
	Rectangular	Polar	Parametric
Parabola	$y = ax^2 + bx + c$ $y = (x - h)^2 + k$ Vertical Axis of Symmetry: $x^2 = 4py$ $(x - h)^2 = 4p(y - k)$ Vertex: (h, k) Focus: $(h, k + p)$ Directrix: $y = k - p$ Horizontal Axis of Symmetry: $y^2 = 4px$ $(y - k)^2 = 4p(x - h)$ Vertex: (h, k) Focus: $(h + p, k)$ Directrix: $x = h - p$ 	Vertical Axis of Symmetry: $r = \frac{2d}{1 + e \cos \theta}$ Eccentricity: $e = 1$ and $d = 2p$ Trigonometric Form: $y = x^2$ $r \sin \theta = r^2 \cos^2 \theta$ $r = \frac{\sin \theta}{\cos^2 \theta} = \tan \theta \sec \theta$  Interesting Note: The distances from a point on the curve to the focus is the <u>same</u> as to the directrix.	Vertical Axis of Symmetry: $x(t) = 2pt + h$ $y(t) = pt^2 + k$ (opens upwards) $y(t) = -pt^2 - k$ (opens downwards) $[t_{\min}, t_{\max}] = [-c, c]$ Vertex: (h, k) Horizontal Axis of Symmetry: $y(t) = 2pt + k$ $x(t) = pt^2 + h$ (opens to the right) $x(t) = -pt^2 - h$ (opens to the left) $[t_{\min}, t_{\max}] = [-c, c]$ Vertex: (h, k) Projectile Motion: $x(t) = x_0 + v_x t + \left(\frac{1}{2}\right) a_x t^2$ $y(t) = y_0 + v_{y0} t - 16t^2 \text{ feet}$ $y(t) = y_0 + v_{y0} t - 4.9t^2 \text{ meters}$ $v_x = v \cos \theta$ $v_y = v \sin \theta$ General Form: $x = At^2 + Bt + C$ $y = Dt^2 + Et + F$ where A and D have the same sign

	Rectangular	Polar	Parametric
1st Derivative	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ $f'(x) = \frac{dy}{dx} = y' = D_x$	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$ <i>Hint: Use Product Rule for</i> $y = r \sin \theta$ $x = r \cos \theta$	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \text{provided } \frac{dx}{dt} \neq 0$
2nd Derivative	$f''(x) = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2 y}{dx^2} = y'' = D_{xx}$	$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{\frac{d}{d\theta} \left(\frac{dy}{dx} \right)}{\frac{dx}{d\theta}}$	$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right)}{\frac{dx}{dt}}$
Integral	<i>Fundamental Theorem of Calculus:</i> $F(x) = \int_a^b f(x) dx = F(b) - F(a)$  <i>Riemann Sum:</i> $S = \sum_{i=1}^n f(y_i)(x_i - x_{i-1})$ <i>Left Sum:</i> $S = \left(\frac{1}{n}\right) \left[f(a) + f\left(a + \frac{1}{n}\right) + f\left(a + \frac{2}{n}\right) + \dots + f\left(b - \frac{1}{n}\right) \right]$ <i>Middle Sum:</i> $S = \left(\frac{1}{n}\right) \left[f\left(a + \frac{1}{2n}\right) + f\left(a + \frac{3}{2n}\right) + \dots + f\left(b - \frac{1}{2n}\right) \right]$ <i>Right Sum:</i> $S = \left(\frac{1}{n}\right) \left[f\left(a + \frac{1}{n}\right) + f\left(a + \frac{2}{n}\right) + \dots + f(b) \right]$		

	Rectangular	Polar	Parametric
Inverse Functions	$f(f^{-1}(x)) = f^{-1}(f(x)) = x$ <i>Inverse Function Theorem:</i> $f^{-1}(f'(a)) = \frac{1}{f'(a)}$	if $y = \sin \theta$ then $\theta = \sin^{-1} y$ if $y = \cos \theta$ then $\theta = \cos^{-1} y$ if $y = \tan \theta$ then $\theta = \tan^{-1} y$ if $y = \csc \theta$ then $\theta = \csc^{-1} y$ if $y = \sec \theta$ then $\theta = \sec^{-1} y$ if $y = \cot \theta$ then $\theta = \cot^{-1} y$	or $\theta = \arcsin y$ or $\theta = \arccos y$ or $\theta = \arctan y$ or $\theta = \operatorname{arccsc} y$ or $\theta = \operatorname{arcsec} y$ or $\theta = \operatorname{arccot} y$
Arc Length	$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$ <i>Proof:</i> $\Delta s = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ $\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $ds = \sqrt{dx^2 + dy^2}$ $ds = \sqrt{dx^2 + dy^2 \left(\frac{dy^2}{dx^2}\right)}$ $ds = \sqrt{dx^2 + \left(\frac{dy}{dx}\right)^2 dx^2}$ $ds = \sqrt{dx^2 \left(1 + \left(\frac{dy}{dx}\right)^2\right)}$ $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ $L = \int ds$	$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ <i>Circle:</i> $L = s = r\theta$ <i>Proof:</i> $L = (\text{fraction of circumference}) \cdot \pi \cdot (\text{diameter})$ $L = \left(\frac{\theta}{2\pi}\right) \pi (2r) = r\theta$  s = rθ	$L = \int_a^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ $L = \int_a^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$ <i>Proof:</i> $ds = \sqrt{dx^2 + dy^2}$ $ds = \sqrt{dx^2 \left(\frac{dt^2}{dt^2}\right) + dy^2 \left(\frac{dt^2}{dt^2}\right)}$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt^2$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ $L = \int ds$
Perimeter	<i>Square:</i> $P = 4s$ <i>Rectangle:</i> $P = 2l + 2w$ <i>Triangle:</i> $P = a + b + c$ <i>Circle:</i> $C = \pi d = 2\pi r$ <i>Ellipse:</i> $C \approx \pi(a + b)$	<i>Ellipse:</i> $C \approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$ $C \approx \pi [3(a + b) - \sqrt{(3a + b)(a + 3b)}]$ $C \approx \pi(a + b) \left(1 + \frac{3h}{10 + \sqrt{4 - 3h}}\right)$	<i>Ellipse:</i> $C = 4a \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2 \theta} d\theta$ $h = \frac{(a - b)^2}{(a + b)^2} \quad \& \quad k^2 = \left(1 - \frac{b^2}{a^2}\right)$

	Rectangular	Polar	Parametric
Area	<i>Square:</i> $A = s^2$ <i>Rectangle:</i> $A = lw$ <i>Rhombus:</i> $A = \frac{1}{2} ab$ <i>Parallelogram:</i> $A = Bh$ <i>Trapezoid:</i> $A = \frac{(B_1 + B_2)}{2} h$ <i>Kite:</i> $A = \frac{d_1 d_2}{2}$ <i>Triangle:</i> $A = \frac{1}{2} Bh$ <i>Triangle:</i> $A = \frac{1}{2} ab \sin(C)$ <i>Triangle using Heron's Formula:</i> $A = \sqrt{s(s-a)(s-b)(s-c)}$ <i>where</i> $s = \frac{a+b+c}{2}$ <i>Equilateral Triangle:</i> $A = \frac{1}{4}\sqrt{3}s^2$ <i>Frustum:</i> $A = \frac{1}{3} \left(\frac{B_1 + B_2}{2} \right) h$ <i>Circle:</i> $A = \pi r^2$ <i>Circular Sector:</i> $A = \frac{1}{2} r^2 \theta$ <i>Ellipse:</i> $A = \pi ab$	$A = \int_{\alpha}^{\beta} \frac{1}{2} [f(\theta)]^2 d\theta$ where $r = f(\theta)$  Area of a sector where arc length $s = r\theta$: $A = \int s dr = \int r\theta dr = \frac{1}{2} r^2 \theta$	$A = \int_{\alpha}^{\beta} g(t) f'(t) dt$ where $f(t) = x$ and $g(t) = y$ or $x(t) = f(t)$ and $y(t) = g(t)$ <i>Simplified:</i> $A = \int_{\alpha}^{\beta} y(t) \frac{dx(t)}{dt} dt$ <i>Proof:</i> $\int_a^b f(x) dx$ $y = f(x) = g(t)$ $dx = \frac{df(t)}{dt} dt = f'(t) dt$
Lateral Surface Area	<i>Cylinder:</i> $SA = 2\pi rh$ <i>Cone:</i> $SA = \pi rl$ $SA = 2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2} dx$	For rotation about the x-axis: $SA = \int 2\pi y ds$ For rotation about the y-axis: $SA = \int 2\pi x ds$ $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ $r = f(\theta), \quad \alpha \leq \theta \leq \beta$	For rotation about the x-axis: $SA = \int 2\pi y ds$ For rotation about the y-axis: $SA = \int 2\pi x ds$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ if $x = f(t), y = g(t), \alpha \leq t \leq \beta$

	Rectangular	Polar	Parametric
Total Surface Area	<i>Cube:</i> $SA = 6s^2$ <i>Rectangular Box:</i> $SA = 2lw + 2wh + 2hl$ <i>Regular Tetrahedron:</i> $SA = 2bh$ <i>Cylinder:</i> $SA = 2\pi r(r + h)$ <i>Cone:</i> $SA = \pi r^2 + \pi rl = \pi r(r + l)$ <i>Sphere:</i> $SA = 4\pi r^2$ <i>Ellipsoid:</i> $SA \approx 4\pi \left(\frac{a^p b^p + a^p c^p + b^p c^p}{3} \right)^{1/p}$ Where $p \approx 1.6075$, $Relative\ Error \leq 1.061\%$ (Knud Thomsen's Formula)		
Surface of Revolution	For revolution about the x-axis: $A = 2\pi \int_a^b f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ For revolution about the y-axis: $A = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$	For revolution about the x-axis: $A = 2\pi r \int_{\alpha}^{\beta} \cos \theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ For revolution about the y-axis: $A = 2\pi r \int_{\alpha}^{\beta} \sin \theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$	For revolution about the x-axis: $A = 2\pi \int_a^b y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ For revolution about the y-axis: $A = 2\pi \int_a^b x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
Volume	<i>Cube:</i> $V = s^3$ <i>Rectangular Prism:</i> $V = lwh$ <i>Cylinder:</i> $V = \pi r^2 h$ <i>Triangular Prism:</i> $V = Bh$ <i>Tetrahedron:</i> $V = \frac{1}{3} Bh$ <i>Pyramid:</i> $V = \frac{1}{3} Bh = \frac{1}{3} lwh$ <i>Cone:</i> $V = \frac{1}{3} Bh = \frac{1}{3} \pi r^2 h$ <i>Sphere:</i> $V = \frac{4}{3} \pi r^3$ <i>Ellipsoid:</i> $V = \frac{4}{3} \pi abc$		

	Rectangular	Polar	Parametric
Volume of Revolution	Disk Method $V = \int_a^b (\text{area of circle}) d(\text{thickness})$ Rotation about the x-axis: $V = \int_a^b \pi [f(x)]^2 dx$ Rotation about the y-axis: $V = \int_c^d \pi x^2 dy$		
	Washer Method Rotation about the x-axis: $V = \int_a^b \pi \{ [f(x)]^2 - [g(x)]^2 \} dx$	$V = V_{\text{Outer Disk}} - V_{\text{Inner Disk}}$	
	Shell Method $V = \int_a^b (\text{circumference}) (\text{height}) dx$ Rotation about the y-axis: $V = \int_a^b 2\pi x f(x) dx$ Rotation about the x-axis: $V = \int_c^d 2\pi y g(y) dy$		

	Rectangular	Polar	Parametric
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