**Harold’s Calculus 3**

**in Multiple Coordinate Systems**

**Cheat Sheet**

15 October 2025

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|  | **Rectangular** | **Polar/Cylindrical** | | | **Spherical** | **Parametric** | **Vector** | **Matrix** |
| **Point** | 2D:  *or*  *3D:*  *4D:*    undefined | *or* | | |  | *Point in Rectangular :*  *, variable, usually time,*  *with 1 degree of freedom (df)* |  |  |
| *Polar 🡪 Rect.* | *Rect. 🡪 Polar*    NOTE:  returns angles in quadrants I & IV only.  +180◦ to move to quadrants II & III. | |
| **Line** | *Slope-Intercept Form:*  *Point-Slope Form:*  *Normal Form:*  *Calculus Form:*  *where*  *Intercept Form:*  *3D:* | [http://upload.wikimedia.org/wikipedia/commons/thumb/7/78/Polar_to_cartesian.svg/250px-Polar_to_cartesian.svg.png](http://en.wikipedia.org/wiki/File:Polar_to_cartesian.svg)  http://upload.wikimedia.org/wikipedia/commons/thumb/b/b7/Cylindrical_Coordinates.svg/190px-Cylindrical_Coordinates.svg.png | | | http://upload.wikimedia.org/wikipedia/commons/thumb/5/51/Spherical_Coordinates_(Colatitude,_Longitude).svg/360px-Spherical_Coordinates_(Colatitude,_Longitude).svg.png | *where* |  |  |
| **Plane** | *Dot Product of Point-Normal Form:*  where:  *is the normal vector*  *General Form:*  *Intercept Form:* | *s* | | |  | *Parametric Form:*  *where:*   * *is a point on the plane.* * *and  are direction vectors on the plane.* * *s and t are parameters that vary over all real numbers.* | *Vector Form:*  *where:*   * *and are given vectors defining the plane* * *is the vector representing the position of an arbitrary (but fixed) point on the plane* | *Point-Normal Form:*  *or*  Normal Vector:  *Points on the plane:*  *Vectors on the plane:* |
| Raw material | |
| **Conics** | *General Equation for All Conics:*  *where:*  *or*  *Note: If , then square hyperbola*  *Rotation:*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *the coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis*  http://www.sensorsmag.com/files/sensor/nodes/2009/6475/Figure9.gif | *General Equation for All Conics:*  *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *= semi-latus rectum*  *or the line segment running from the focus to the curve in a direction parallel to the directrix*  *Eccentricity:*  Image result for conics | | | A different shapes of geometric shapes  Description automatically generated with medium confidence  Image result for conics | | | *Quadratic Form:*  *Matrix Form:*  *where* |
| **Circle** | *General Form:*  *Center:*  *Vertices: NA*  *Focus:*  Equation of a Circle | *Centered at Origin:*  *r = a (constant)*  *Centered at :*  *Hint: Law of Cosines*  *or*  Image result for off center circle in polar coordinates | |  | | *Center:*  *Focus:* | 2D:  **x** = vector of the points of the circle  **m** = vector to the center of the circle  r = radius  3D:  center of the circle  are two orthogonal unit vectors in the plane of the circle | *where* |
| **Sphere** | *General Form:*  > 0  *Cylindrical to Rectangular:*  Spherical to Rectangular: | *Rectangular to Cylindrical:*  *Spherical to Cylindrical:* | | *Rectangular to Spherical:*  *Cylindrical to Spherical:* | | [https://encrypted-tbn1.gstatic.com/images?q=tbn:ANd9GcQRFtXKyazGXHqZX_E5FPf22QXFOrEkEa0c2XZ6MTKC3ATaXwJW3w](http://www.google.com/imgres?safe=off&sa=N&hl=en&biw=1287&bih=815&tbm=isch&tbnid=D5odqmhVPnxFGM:&imgrefurl=http://maverick.inria.fr/~Xavier.Decoret/resources/xdkwrl/html/a00281.html&docid=9f5rjsFcwRafOM&imgurl=http://maverick.inria.fr/~Xavier.Decoret/resources/xdkwrl/html/images/sphere.gif&w=342&h=334&ei=W692UdyrBsr32wWR3IDQBg&zoom=1&ved=1t:3588,r:38,s:0,i:276&iact=rc&dur=946&page=2&tbnh=177&tbnw=181&start=25&ndsp=30&tx=94&ty=84) | Rectangular:  Cylindrical:  Spherical: | [https://encrypted-tbn3.gstatic.com/images?q=tbn:ANd9GcTcVzEz8siGxLf9_AvM0HFSxumP4P2eM0hQ0AIpN120-0q5Y7TC-Q](http://www.google.com/imgres?safe=off&sa=N&hl=en&biw=1287&bih=815&tbm=isch&tbnid=X5STfIPBciE8CM:&imgrefurl=http://library.thinkquest.org/20991/geo/solids.html&docid=eb6vwUkIGv-EiM&imgurl=http://library.thinkquest.org/20991/media/geo_sphere.gif&w=300&h=300&ei=W692UdyrBsr32wWR3IDQBg&zoom=1&ved=1t:3588,r:13,s:0,i:191&iact=rc&dur=773&page=1&tbnh=177&tbnw=189&start=0&ndsp=25&tx=135&ty=41) |
| **Ellipse** | *General Form:*  *where*  *Center:*  *Vertices:*  *Co-Vertices:*  *Foci:*  *Focus length, c, from center:*  *Eccentricity:*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis* | *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *relative to center*  http://newportaoit.org/tfuentes/ellipse2.gif | | Image result for conics  See the source image  ***Interesting Note:***  *The sum of the distances from each focus to a point on the curve is constant.* | | *Center:*  *Rotated Ellipse:*  *= the angle between the x-axis and the major axis of the ellipse*  http://www.sensorsmag.com/files/sensor/nodes/2009/6475/Figure9.gif | center of the ellipse  are two orthogonal unit vectors in the plane of the ellipse | *where* |
| **Ellipsoid** |  |  | |  | |  | http://upload.wikimedia.org/wikipedia/commons/thumb/5/50/Ellipsoid_tri-axial_abc.svg/200px-Ellipsoid_tri-axial_abc.svg.png | *Centered at vector* |
| **Hyperbola** | *General Form:*  *where*  *If , square hyperbola*  *Center:*  *Vertices:*  *Foci:*  *Focus length, c, from center:*  *Eccentricity:*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis* | Hyperbola  ***Interesting Note:***  *The difference between the distances from each focus to a point on the curve is constant.* | | *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *relative to center*  Image result for "latus rectum" of a hyperbola  *p = semi-latus rectum*  *or the line segment running from the focus to the curve in the directions* | | *Left-Right Opening Hyperbola:*  *Vertex: (h, k)*  *Alternate Form:*  *Up-Down Opening Hyperbola:*  *Vertex: (h, k)*  *Alternate Form:*  *General Form:*  *where A and D have different signs* |  | *where* |
| **Hyperboloid** |  |  | | calculus - Finding the vertex of a two-sheet-hyperboloid - Mathematics  Stack Exchange | | |  |  |
| **Parabola** | *Vertical Axis of Symmetry:*  *Vertex:*  *Focus:*  *Directrix:*  *Horizontal Axis of Symmetry:*  *Vertex:*  *Focus:*  *Directrix:*  *General Form:*  *where*  *or*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis* | *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *and*  Parabola | | Image result for conics parabola rectum  ***Interesting Note:***  *The distances from a point on the curve to the focus is the same as to the directrix.* | | *Vertical Axis of Symmetry:*  *(opens upwards)*  *(opens downwards)*  *Vertex:*  *Horizontal Axis of Symmetry:*  *(opens to the right)*  *(opens to the left)*  *Vertex:*  *Projectile Motion:*  *feet*  *meters*  *General Form:*  *where A and L have the same sign* |  | *where* |
| **Paraboloid** |  | Paraboloid – Wikipedia | |  | |  |  |  |

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| **Limit** |  | http://upload.wikimedia.org/wikipedia/commons/thumb/d/d1/L%C3%ADmite_01.svg/200px-L%C3%ADmite_01.svg.png | | http://upload.wikimedia.org/wikipedia/commons/thumb/6/66/Limit-at-infinity-graph.png/306px-Limit-at-infinity-graph.png |  |  |  |
| **1st Derivative** |  | *Hint: Use Product Rule for* | |  |  | *Unit tangent vector* |  |
| **2nd Derivative** |  |  | |  |  | *Unit normal vector* |  |
| **Integral** | *Fundamental Theorem of Calculus:* | [http://upload.wikimedia.org/wikipedia/commons/thumb/2/2a/Riemann_sum_convergence.png/250px-Riemann_sum_convergence.png](http://en.wikipedia.org/wiki/File:Riemann_sum_convergence.png) | |  | *Riemann Sum:*  *Left Sum:*  *Middle Sum:*  *Right Sum:* |  | |
| **Double Integral** |  |  | |  |  |  |  |
| **Triple Integral** |  |  | |  |  |  |  |
| **Inverse Functions** | *Inverse Function Theorem:* | *if*  *if*  *if*  *if*  *if*  *if* |  | *or*  *or*  *or*  *or*  *or*  *or* |  |  |  |
| **Arc Length** | *Proof:* | *Polar:*  *Where*  *Circle:*  *Proof:* | | *C = πd = 2πr*  http://www.mathwarehouse.com/trigonometry/radians/images/picture-s=r-theta-circle.gif | *Rectangular 2D:*  *Rectangular 3D:*  *Cylindrical:*  *Spherical:* | *s(t)* |  |
| **Curvature** |  | *for r()* | |  | *where f(t) = (x(t), y(t), z(t))* |  | *(See Wikipedia:* [*Curvature*](https://en.wikipedia.org/wiki/Curvature)*)* |
| **Perimeter** | *Square:* *P = 4s*  *Rectangle:* *P = 2l + 2w*  *Triangle:* *P = a + b + c*  *Circle:* *C = πd = 2πr*  *Ellipse:* | *Ellipse:* | | *Ellipse:* |  |  |  |
| **Area** | *Square:* *A = s²*  *Rectangle:* *A = lw*  *Rhombus:* *A = ½ ab*  *Parallelogram:* *A = Bh*  *Trapezoid:*  *Kite:*  *Triangle:* *A = ½ Bh*  *Triangle:* *A = ½ ab sin(C)*  *Triangle using Heron’s Formula:*  *Equilateral Triangle:*  *Frustum:*  *Circle:* *A = πr²*  *Circular Sector:* *A = ½ r²*  *Ellipse:* *A = πab* | *where*  [http://upload.wikimedia.org/wikipedia/commons/thumb/4/4c/Polar_coordinates_integration_Riemann_sum.svg/220px-Polar_coordinates_integration_Riemann_sum.svg.png](http://en.wikipedia.org/wiki/File:Polar_coordinates_integration_Riemann_sum.svg)  *Area of a sector where arc length :* | |  | *where and*  *or*  *x(t) = f(t) and y(t) = g(t)*  *Simplified:*  *Proof:*  *y = f(x) = g(t)* |  |  |
| **Lateral Surface Area** | *Cylinder:* *SA = 2πrh*  *Cone:* *SA = πrl* | *For rotation about the x-axis:*  *For rotation about the y-axis:* | | *Sphere:* | *For rotation about the x-axis:*  *For rotation about the y-axis:* |  |  |
| **Total Surface Area** | *Cube:* *SA = 6s²*  *Rectangular Box: SA = 2lw + 2wh + 2hl*  *Regular Tetrahedron: SA = 2bh*  *Cylinder:* *SA = 2πr (r + h)*  *Cone:* *SA = πr² + πrl = πr (r + l)*  *Sphere:* *SA = 4πr²* | *Ellipsoid: SA*  *Where p*  *(Knud Thomsen’s Formula)* | | *Ellipsoid: S =* | http://www.numericana.com/answer/G.1.7.xml_gr_1.gif  *where* http://www.numericana.com/answer/G.1.7.xml_gr_2.gif | | |
| **Surface of Revolution** | *For revolution about the x-axis:*  *For revolution about the y-axis:* | *For revolution about the x-axis:*  *For revolution about the y-axis:* | | *Sphere: S = 4πr²* | *For revolution about the x-axis:*  *For revolution about the y-axis:* |  |  |
| **Volume** | *Cube:*  *Rectangular Prism:*  *Cylinder:*  *Triangular Prism:*  *Tetrahedron:*  *Pyramid:*    *Cone:*    S*phere:* *Ellipsoid:* |  | |  |  | The volume element in spherical coordinates | *Ellipsoid:* |

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| **Volume of Revolution** | **Disk Method**  *Rotation about the x-axis:*  *Rotation about the y-axis:* | cochranmath / Volume of a solid of revolution by plane slicing | 1.1: Volumes of solids of revolution -cross-sections - Mathematics  LibreTexts |  |  |
| **Washer Method**  *Rotation about the x-axis:* | (Disk Method twice) |  |  |  |
| **Shell Method**  *Rotation about the y-axis:*  *Rotation about the x-axis:* | 2.3 Volumes of Revolution: Cylindrical Shells - Calculus Volume 2 ... | This figure has two images. The first is labeled “a” and is of a hollow cylinder around the y-axis. On the front of this cylinder is a vertical line labeled “cut line”. The height of the cylinder is “y=f(x)”. The second figure is labeled “b” and is a shaded rectangular block. The height of the rectangle is “f(x*), the width of the rectangle is “2pix*”, and the thickness of the rectangle is “delta x”. | |  |

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| **Moment of Inertia** |  |  |  | Derivation Of Moment Of Inertia Of An Uniform Rigid Rod | Mini Physics -  Free Physics Notes |  | *(see Wikipedia:* [*Moment of Inertia*](https://en.wikipedia.org/wiki/Moment_of_inertia#Motion_in_space_of_a_rigid_body,_and_the_inertia_matrix)*)* |
| **Center of Mass** | *1D for Discrete:*  *where* | *2D for Discrete:* | *3D for Discrete:* | *3D for Continuous:*  *where*  *and* | *where is distance from the axis of rotation, not origin.* |  |
| **Gradient** |  |  |  | http://upload.wikimedia.org/wikipedia/commons/thumb/3/31/Gradient99.png/350px-Gradient99.png | *where* | enter image description here |
| **Line Integral**  (Contour Integral if Complex) |  |  |  |  | *Fundamental Theorem for Line Integrals* | |
| **Surface Integral** | *Where*  *and* | $$\hbox{\epsfysize=1.75in \epsffile{surface-integrals-1.eps}}$$ | $$\hbox{\epsfysize=1.75in \epsffile{surface-integrals-2.eps}}$$ | http://upload.wikimedia.org/wikipedia/commons/thumb/8/87/Surface_integral1.svg/220px-Surface_integral1.svg.png |  | Vector surface integral examples - Math Insight |