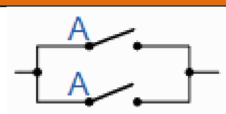
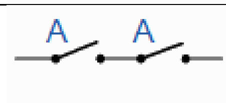
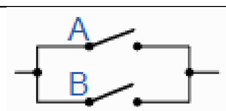
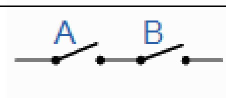
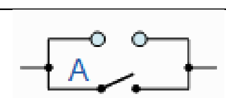
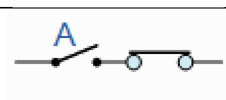
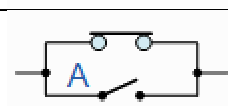
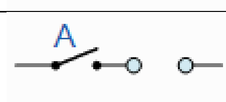
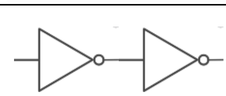
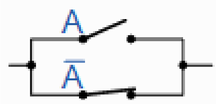
Harold's Boolean Algebra
Cheat Sheet
8 September 2025

Boolean Algebra

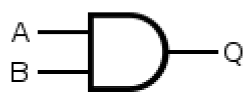
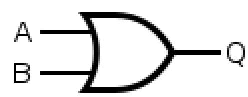
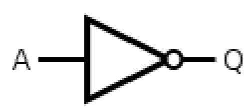
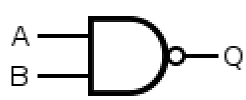
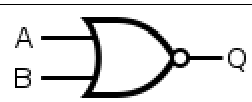
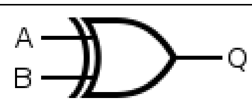
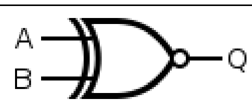
Boolean Law	Boolean Expression	Equivalent Circuit	Description
Idempotent	$A + A = A$		A in parallel with A = "A"
	$A \bullet A = A$		A in series with A = "A"
Associative	$A + (B + C)$ $= (A + B) + C$ $= A + B + C$		Allows the removal of brackets from an expression and the regrouping of the variables
	$A \bullet (B \bullet C)$ $= (A \bullet B) \bullet C$ $= A \bullet B \bullet C$		
Commutative	$A + B = B + A$		A in parallel with B = B in parallel with A
	$A \bullet B = B \bullet A$		A in series with B = B in series with A
Distributive	$A \bullet (B + C)$ $= A \bullet B + A \bullet C$		Permits the multiplying or factoring out of an expression
	$A + B \bullet C =$ $(A + B) \bullet (A + C)$		
Identity	$A + 0 = A$		A in parallel with open = "A"
	$A \bullet 1 = A$		A in series with closed = "A"
Domination (Annulment)	$A + 1 = 1$		A in parallel with closed = "CLOSED"
	$A \bullet 0 = 0$		A in series with open = "OPEN"
Double Compliment (Double Negation)	$\overline{\overline{A}} = A$		NOT NOT A (double negative) = "A"

Complement	$A + \overline{A} = 1$		A in parallel with NOT A = “CLOSED”
	$A \bullet \overline{A} = 0$		A in series with NOT A = “OPEN”
De Morgan’s	$\overline{(A + B)} = \overline{A} \bullet \overline{B}$		Invert and replace OR with AND
	$\overline{A \bullet B} = \overline{A} + \overline{B}$		Invert and replace AND with OR
Absorption (Absorptive)	$A + (AB) = A$		Enables a reduction in a complicated expression to a simpler one by absorbing like terms
	$A(A + B) = A$		
	$A + \overline{A}B = A + B$		Reduces a complicated expression to a simpler one by absorbing complement term

Precedence Rules

#	Operator	Symbol	Precedence
1	**Parenthesis**	( )	Highest precedence
2	**NOT**	~	
3	**Quantifiers**	∀, ∃	
4	**AND**	•	Applied Left to Right
5	**OR**	∨	
6	**Conditional**	⊃	
7	**Biconditional**	↔	Lowest precedence

Boolean Logic Gates

Boolean Logic	Notation	Gate	Description
IDENTITY	1 T True S		On, Tautology, High voltage (typically +5V)
NULL	0 F False ⊥ Ø	**GND** 	Off, Contradiction, Low voltage (typically 0V)
Input	A, B, C, D		Line, Wire, Connects to
Output	W, X, Y, Z		Line, Wire, Connects from
AND	$A \cdot B$ AB $A. B$ $A \wedge B$ $A \cap B$		AND, BUT, Multiply, Conjunction, Intersection
OR	$A + B$ $A \vee B$ $A \cup B$ A | B		Inclusive-OR, Add, Disjunction, Union
NOT	$\overline{A}$ $A^\wedge$ A' $\neg A$ $\sim A$ $!A$		NOT, Invert, Negation, Change, Difference
NAND	$\overline{A \cdot B}$ $\overline{AB}$ $A \barwedge B$ A | B*		Not AND
NOR	$\overline{A + B}$ $A \overline{\vee} B$ $A \downarrow B$		Not OR
XOR	$A \oplus B$ $A \veebar B$ $A\overline{B} + \overline{A}B$		Exclusive-OR, Both A and B are different
XNOR	$A \odot B$ $\overline{A \oplus B}$ $AB + \overline{AB}$		Exclusive-NOR, Both A and B are the same

NOTE: The laws of Boolean algebra are the same in propositional and set logic.

Boolean Logic Truth Tables

Inputs		Outputs								
A	B	AND $\cdot$	NAND $\bar{\wedge}$	OR +	NOR $\bar{\vee}$	XOR $\oplus$	XNOR $\odot$	NOT $\bar{A}$	VCC 1	GND 0
0	0	0	1	0	1	0	1	A=1	1	0
0	1	0	1	1	0	1	0	A=1	1	0
1	0	0	1	1	0	1	0	A=0	1	0
1	1	1	0	1	0	0	1	A=0	1	0

Blank Truth Tables

Inputs		Output
A	B	X
0	0	
0	1	
1	0	
1	1	

Inputs			Output	
A	B	C	X	Y
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Inputs				Output		
A	B	C	D	X	Y	Z
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			

Karnaugh Mapping (K-Map)

2-Bit K-Map		**A**	
		0	1
B	0		
	1		

3-Bit K-Map		**AB**			
		00	01	11	10
C	0				
	1				

4-Bit K-Map		**AB**			
		00	01	11	10
CD	00				
	01				
	11				
	10				

2x2 Group

1x4 Group

K-Map Rules

1) Circle only 1s (ones) and don't cares for Sum of Products (SOP), $e.g.\ \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC + AB\overline{C}$.
    a. Circle only 0s (zeros) and don't cares for Product of Sums (POS), $e.g.\ (A + \overline{B})(\overline{A} + B)$.
    b. Don't cares may be used or ignored.
2) No diagonals, only horizontal or vertical connections.
3) Group only adjacent cells in groups with powers of 2 (1x1, 1x2, 2x1, 2x2, 2x4, 4x2, 1x4, 4x1).
4) Make groups as large as possible.
5) Must group all 1s (ones) for SOP or all 0s (zeros) for POS.
6) Overlapping is allowed.
7) Wrapping around all edges allowed, both top-bottom edges and left-right edges.
8) Fewest groups possible (OPTIMAL).
9) For each circle, determine which inputs do not contribute to the logic (is both 0 and 1).
10) Write down the equation as a SOP, $e.g.\ \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC + AB\overline{C}$

Sources

- SNHU MAT 230 - Discrete Mathematics, zyBooks.
- https://www.electronics-tutorials.ws/boolean/bool_6.html
- Toomey, H. A. (2024). Logical Connective Laws, Harold's Logic Cheat Sheet, https://www.toomey.org/tutor/discrete_math.html

See Also

- Harold's Logic Cheat Sheet
- Harold's Logic (Philosophy) Cheat Sheet
- Harold's Sets Cheat Sheet
- Harold's Boolean Algebra Cheat Sheet
- Harold's Proofs Cheat Sheet

