

AP[®] PHYSICS C: MECHANICS

2009 SCORING GUIDELINES

General Notes About 2009 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point and a student's solution contains the application of that equation to the problem, but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics Exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections—Student Presentation” in the *AP Physics Course Description*.
4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

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Question 2

15 points total

Distribution of points

(a)

(i) 4 points

For the rotational form of Newton's second law

1 point

$$\tau = I\alpha$$

For a correct expression of the magnitude of torque

1 point

For correctly labeling the torque as negative

1 point

$$-Mgx \sin \theta = I_b \alpha$$

For expressing α as the second time derivative of θ

1 point

$$-Mgx \sin \theta = I_b \left(\frac{d^2 \theta}{dt^2} \right)$$

(ii) 4 points

For the appropriate small angle approximation

1 point

For small angles, $\sin \theta \approx \theta$

$$-Mgx\theta = I_b \left(\frac{d^2 \theta}{dt^2} \right)$$

$$\left(\frac{d^2 \theta}{dt^2} \right) + \left(\frac{Mgx}{I_b} \right) \theta = 0$$

For recognizing that the coefficient of θ is ω^2

1 point

$$\omega^2 = \frac{Mgx}{I_b}$$

For the relationship between T and ω (this point was awarded for the equation alone or with relevant work, but NOT as part of multiple random equations)

1 point

$$T = \frac{2\pi}{\omega}$$

For the final expression for T (this point was awarded for the final correct answer with no supporting work)

1 point

$$T = 2\pi \sqrt{\frac{I_b}{Mgx}}$$

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Question 2 (continued)

Distribution of points

(b) 5 points

For an experimental procedure that includes:

- | | |
|--|----------|
| A valid approach | 1 point |
| How the variables will be measured or calculated, including equipment to be used | 2 points |
| How these variables will be used to determine I_B | 1 point |
| How to minimize error | 1 point |

Example 1: Displace the bar by a small angle and release it to oscillate. To reduce errors, time 10 complete oscillations with a stopwatch. Calculate the average value of the time for 10 oscillations and then divide by 10 to determine the period T .

Calculate I_B from $T = 2\pi\sqrt{I_b/Mgx}$, using known values of M and x .

Example 2: Locate a photogate at the bottom of the bar's swing; set it to measure the amount of time the photogate is blocked. While the bar is hanging from its pivot point, displace the bar to a horizontal position and measure the height of the center of mass above the position of the photogate with a meter stick. Allow the bar to swing through the photogate and obtain the time the gate is blocked. To reduce errors, repeat this procedure five times and obtain an average time. Measure the width of the bar and use this and the time to determine the speed of the bar at the bottom of the swing, $v = \text{width}/\text{time}$. Calculate the angular speed of the bar from

$\omega = v/\ell$. Apply conservation of energy to the bar: $Mgh = I_B\omega^2/2$. Calculate I_B

from $I_B = 2Mgh/\omega^2$, using known values of M , measured value of h , and calculated value of ω .

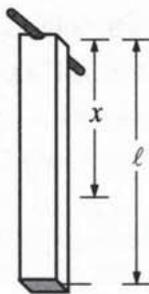
(c) 2 points

For a valid procedure to locate the center of mass	1 point
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For specifying the equipment to be used	1 point
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Example 1: Place the bar on top of a fulcrum, e.g., the top of a prism. Adjust the position of the bar until it is balanced horizontally. The point at which this occurs is the center of mass.

Example 2: Place the bar near the edge of a desk or table. Slowly push the bar so it hangs off the table until it is just ready to tip. The point at which this occurs is the center of mass.



Mech. 2.

You are given a long, thin, rectangular bar of known mass M and length l with a pivot attached to one end. The bar has a nonuniform mass density, and the center of mass is located a known distance x from the end with the pivot. You are to determine the rotational inertia I_b of the bar about the pivot by suspending the bar from the pivot, as shown above, and allowing it to swing. Express all algebraic answers in terms of I_b , the given quantities, and fundamental constants.

(a)

- i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle θ the bar makes with the vertical.

$\sum \tau = \sum \vec{F} \times \vec{r} = -F_{grav} \times \sin \theta = -Mg \times \sin \theta$ (cross product)
 $\sum \tau = I_b \alpha \quad -Mg \times \sin \theta = I_b \alpha$
 $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
 $\alpha = \frac{Mg}{I_b} \times \sin \theta = -\alpha$
 $\frac{d^2\theta}{dt^2} = \frac{-Mg}{I_b} \sin \theta$

- ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

small-angle approx: for small θ , $\sin \theta \approx \theta$.

Thus: $\frac{d^2\theta}{dt^2} = \frac{-Mg}{I_b} \theta$. This describes simple harmonic motion

$$\frac{d^2\theta}{dt^2} = -\omega^2 \theta$$

Thus $\omega^2 = \frac{Mg}{I_b}$ $\omega = \sqrt{\frac{Mg}{I_b}}$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{Mg}{I_b}}}$

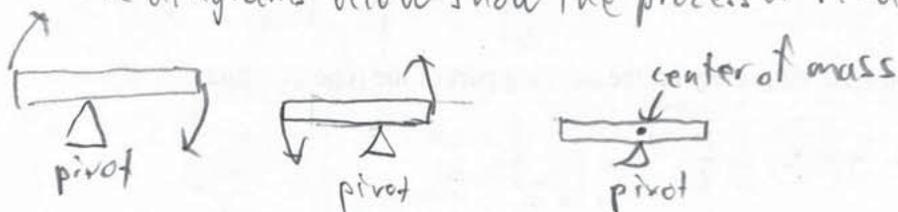
$$T = 2\pi \sqrt{\frac{I_b}{Mg}}$$

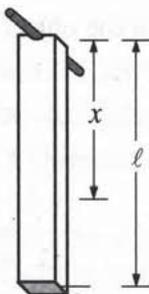
- (b) Describe the experimental procedure you would use to make the additional measurements needed to determine I_b . Include how you would use your measurements to obtain I_b and how you would minimize experimental error. To determine I_b , I would first have to determine T .

I would do this by pulling the bar horizontally so it makes a small angle θ with the vertical, then releasing it. I would use a stopwatch to time 10 swings, then divide by 10 to get a value for T (using 10 swings reduces measuring error). Knowing T , I could calculate I_b from the relation $T = 2\pi \sqrt{\frac{I_b}{Mgx}}$ found in (a-ii), since that formula is equivalent to: $I_b = Mgx \left(\frac{T}{2\pi}\right)^2$.

- (c) Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.

When an object is balanced on a pivot at its center of mass, it will remain in equilibrium. Therefore, I would require a pivot/fulcrum to find the center of mass. I would balance the bar on the pivot repeatedly, adjusting the position each time until a point is found on which the bar balances perfectly and does not tip. That point is the location of the center of mass. The diagrams below show the process of finding the center of mass.





Mech. 2.

You are given a long, thin, rectangular bar of known mass M and length l with a pivot attached to one end. The bar has a nonuniform mass density, and the center of mass is located a known distance x from the end with the pivot. You are to determine the rotational inertia I_b of the bar about the pivot by suspending the bar from the pivot, as shown above, and allowing it to swing. Express all algebraic answers in terms of I_b , the given quantities, and fundamental constants.

(a)

- i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle θ the bar makes with the vertical.

$$\omega = \frac{d\theta}{dt}$$

$$\frac{xg}{l} = \frac{d\theta}{dt}$$

$$\boxed{d\theta = \frac{xg}{l} dt}$$

$$\vec{\tau}_\theta = I \alpha = M \times g = \frac{m v^2}{l}$$

$$v = \omega l$$

$$M \times g = m \omega l$$

$$\omega = \frac{xg}{l}$$

- ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{d\theta}{dt}} = \frac{2\pi}{\pi/2} = \boxed{4A = T}$$

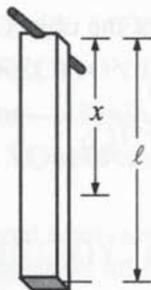
$$\theta \approx \pi/2$$

- (b) Describe the experimental procedure you would use to make the additional measurements needed to determine I_b . Include how you would use your measurements to obtain I_b and how you would minimize experimental error.

To determine I_b , you would need to time with some sort of stop watch or GLX, how long it takes to go through one period of motion, giving you the period, which would enable you to calculate I_b using the center of mass, period, and angle. To minimize experimental error time how long it takes for the bar to complete multiple oscillations and then divide by that number, giving you a more accurate period, and therefore a more accurate measure of I_b .

- (c) Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.

To find the center of mass of the bar, you would need a meter stick, a balancing apparatus, and a scale. By testing points on the bar trying to get the bar to stabilize at equilibrium, you will be able to measure where the center of mass is with the meter stick.



Mech. 2.

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(a)

- i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle θ the bar makes with the vertical.

$$\tau = I\alpha \quad a = ar \quad \alpha = \frac{a}{r} = \frac{d^2\theta}{dt^2} = \omega^2$$

$$mgsin\theta r = I\alpha$$

$$mgsin\theta r = I \frac{d^2\theta}{dt^2}$$

$$\frac{mgsin\theta r}{I} = \omega^2$$

- ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

$$\sin\theta \approx \theta \quad \sin 90^\circ \approx 1$$

$$\omega = \sqrt{\frac{mgr}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{mgr}} = \boxed{2\pi \sqrt{\frac{I_b}{Mgl}}}$$

- (b) Describe the experimental procedure you would use to make the additional measurements needed to determine I_b . Include how you would use your measurements to obtain I_b and how you would minimize experimental error.

In order to determine I_b , I would need to measure the mass of the rectangular bar as well as its length. The final piece of data I would need is the period of one revolution for the bar. In order to calculate this, I would use a stopwatch to find the ^{amount of} time the bar takes to go from a horizontal position to a vertical position (x) from the center of mass. To minimize error,

- (c) Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.

To determine the center of mass, I would need a stopwatch, a meterstick, and a balance. First, I would use the formula for center of mass in both the x and y directions to find the point on the rectangular bar that it would be. Then I would locate the point using a meterstick and from there, I could perform the same experiment as above.

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2009 SCORING COMMENTARY

Question 2

Overview

Part (a) of this question assessed students' ability to analyze torques acting on a rigid swinging bar having a nonuniform mass density and pivoted at one end. Students were asked to apply the appropriate equation of motion to the bar, to write the differential equation for the angle the bar makes with the vertical, and, from their differential equation, to calculate the period of the bar's motion.

Parts (b) and (c) assessed students' ability to write an experimental procedure. Part (b) asked students to describe an experimental procedure they would use to take the additional measurements needed to determine the moment of inertia of the bar, including how the measurements would be used to obtain the moment of inertia and how to minimize experimental error. In part (c) students were not given the bar's center of mass, and they were asked to describe an experimental procedure, including the necessary equipment, that could be used to determine it.

Sample: CM-2A

Score: 15

This response represents a complete and clearly stated solution, earning the maximum points for all parts of the problem.

Sample: CM-2B

Score: 8

Part (a)(i) earned 1 point for stating the rotational form of Newton's second law. Part (a)(ii) earned 1 point for the correct relationship between T and ω . Part (b) earned 4 out of 5 points, losing 1 point for failing to correctly explain how I_b would be calculated from the measured quantities. Part (c) earned both points.

Sample: CM-2C

Score: 5

Part (a)(i) earned 1 point for invoking the angular form of Newton's second law. Part (a)(ii) earned 3 points: 1 point for applying the correct small-angle approximation, 1 point for recognition that ω^2 is the coefficient of the θ term, and 1 point for the correct relationship between T and ω . Part (b) earned 1 point for indicating the measurement of the period would be a valid approach; the description of how the period would be measured was not acceptable as the bar is initially placed in the horizontal position. Part (c) earned no credit.