



AP Physics C: Electricity and Magnetism 2000 Student Samples

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(b) Determine the following in terms of the given quantities and fundamental constants.

AAZ

i. The potential difference across the capacitor

$$V = \int E \cdot dl$$

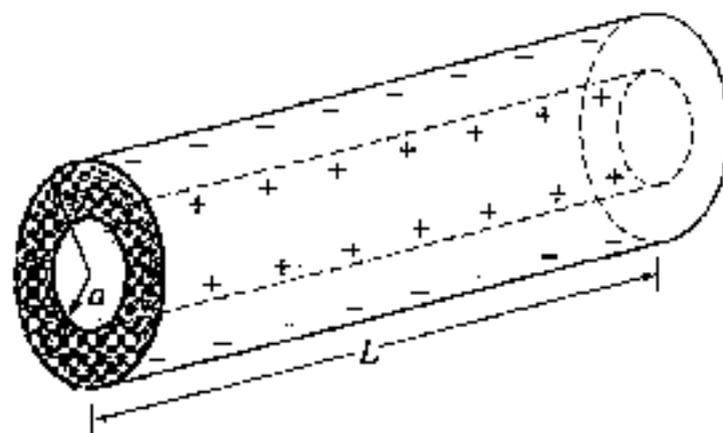
$$V = \int_a^b E(r) dr = \int_a^b \frac{Q}{2\pi\epsilon_0 r L \kappa} dr$$

$$V = \frac{Q \ln(r)}{2\pi\epsilon_0 L \kappa} \Big|_a^b = \boxed{\frac{Q \ln\left(\frac{b}{a}\right)}{2\pi\epsilon_0 L \kappa}}$$

ii. The capacitance of this capacitor

$$Q = CV \quad C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{Q \ln\left(\frac{b}{a}\right)}{2\pi\epsilon_0 L \kappa}} = \boxed{\frac{2\pi\epsilon_0 L \kappa}{\ln\left(\frac{b}{a}\right)}}$$



E & M 3.

A capacitor consists of two conducting, coaxial, cylindrical shells of radius a and b , respectively, and length $L \gg b$. The space between the cylinders is filled with oil that has a dielectric constant κ . Initially both cylinders are uncharged, but then a battery is used to charge the capacitor, leaving a charge $+Q$ on the inner cylinder and $-Q$ on the outer cylinder, as shown above. Let r be the radial distance from the axis of the capacitor.

(a) Using Gauss's law, determine the electric field midway along the length of the cylinder for the following values of r , in terms of the given quantities and fundamental constants. Assume end effects are negligible.

i. $a < r < b$

$$\int E \cdot dA = \frac{q_{in}}{\epsilon_r}$$

$$E \cdot 2\pi r L = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi \epsilon_0 r L}$$

ii. $b < r \ll L$

$$\int E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

Q and $-Q$ cancel, $q_{in} = 0$

$$\int E \cdot dA = 0$$

$$E = 0$$

(b) Determine the following in terms of the given quantities and fundamental constants.

i. The potential difference across the capacitor

$$V_0 = \int_a^b E ds = \int_a^b \frac{Q dr}{2\pi\epsilon_0 r} = \frac{Q}{2\pi\epsilon_0} \int_a^b \frac{dr}{r} = \frac{Q}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

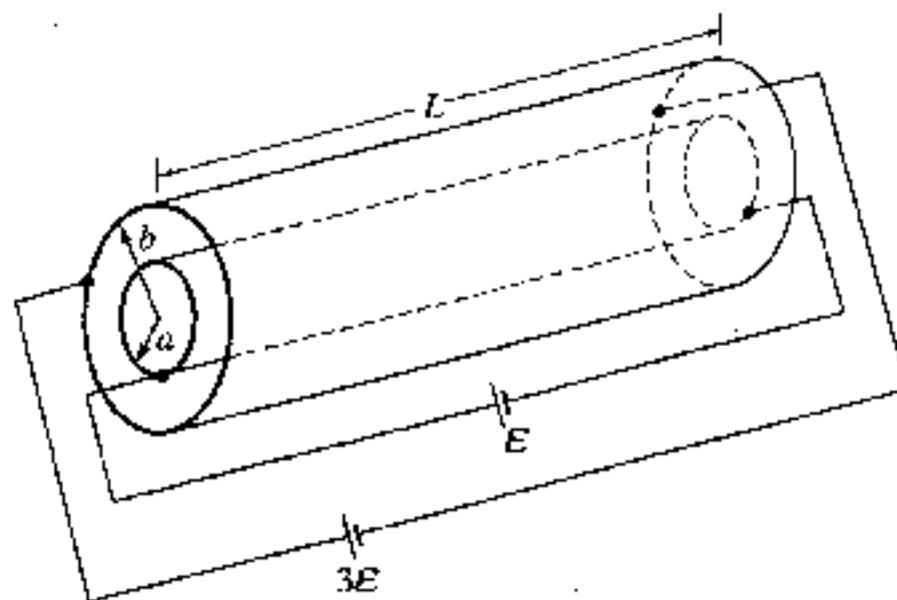
$$V = \frac{V_0}{K} \qquad V = \frac{Q \ln\left(\frac{b}{a}\right)}{2\pi\epsilon_0 K}$$

ii. The capacitance of this capacitor

$$C = \frac{Q}{V} \text{ or } K \frac{Q}{V_0}$$

assuming $V = \frac{Q}{2\pi\epsilon_0 K} \ln\left(\frac{b}{a}\right)$

$$C = \frac{2\pi\epsilon_0 K}{\ln\left(\frac{b}{a}\right)}$$



(c) Now the capacitor is discharged and the oil is drained from it. As shown above, a battery of emf \mathcal{E} is connected to opposite ends of the inner cylinder and a battery of emf $3\mathcal{E}$ is connected to opposite ends of the outer cylinder. Each cylinder has resistance R . Assume that end effects and the contributions to the magnetic field from the wires are negligible. Using Ampere's law, determine the magnitude B of the magnetic field midway along the length of the cylinders due to the current in the cylinders for the following values of r .

i. $a < r < b$

$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_m$$

$$B 2\pi r = \mu_0 \frac{\mathcal{E}}{R}$$

$$B = \frac{\mu_0}{2\pi r} \cdot \frac{\mathcal{E}}{R}$$

ii. $b < r \ll L$

$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_m$$

$$B 2\pi r = \mu_0 \left(\frac{\mathcal{E}}{R} + \frac{3\mathcal{E}}{R} \right)$$

$$B = \frac{2\mu_0}{\pi r} \cdot \frac{\mathcal{E}}{R}$$

(b) Determine the following in terms of the given quantities and fundamental constants.

i. The potential difference across the capacitor

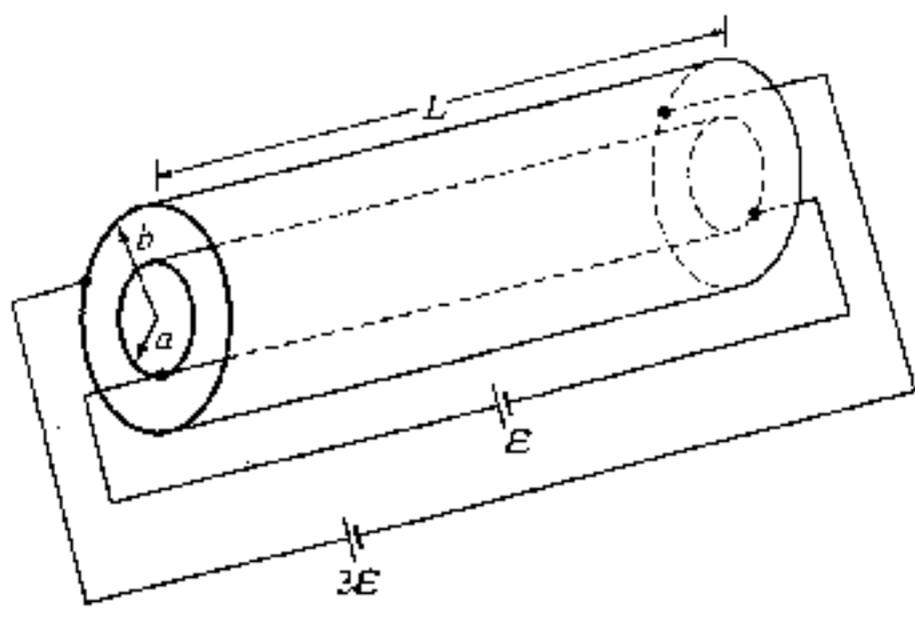
$$\begin{aligned}
 V &= - \int \frac{dE}{dr} = - \frac{Q}{\epsilon_0} \frac{d}{dr} \left[(2\pi r^2 + 2\pi rL)^{-1/2} \right] \\
 &= - \frac{Q}{\epsilon_0} \left(- (2\pi r^2 + 2\pi rL)^{-3/2} (4\pi r + 2\pi L) \right) \\
 &= \frac{Q(4\pi r + 2\pi L)}{\epsilon_0 (2\pi r^2 + 2\pi rL)^{3/2}}
 \end{aligned}$$

ii. The capacitance of this capacitor

$$C = \frac{KQ}{V} = \frac{KQ(\epsilon_0(2\pi r^2 + 2\pi rL)^{3/2})}{Q(4\pi r + 2\pi L)}$$

$$= \frac{K\epsilon_0(2\pi r^2 + 2\pi rL)^{3/2}}{4\pi r + 2\pi L}$$

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(c) Now the capacitor is discharged and the oil is drained from it. As shown above, a battery of emf \mathcal{E} is connected to opposite ends of the inner cylinder and a battery of emf $3\mathcal{E}$ is connected to opposite ends of the outer cylinder. Each cylinder has resistance R . Assume that end effects and the contributions to the magnetic field from the wires are negligible. Using Ampere's law, determine the magnitude B of the magnetic field midway along the length of the cylinders due to the current in the cylinders for the following values of r .

i. $a < r < b$

$$\int B \, dl = \mu_0 I_c$$

$$B = \frac{\mu_0 I_c}{l} = \frac{\mu_0 \mathcal{E}}{R(2\pi r)}$$

$$I_c = \frac{\mathcal{V}}{R}$$

ii. $b < r < l$

$$\int B \, dl = \mu_0 I_c$$

$$B = \frac{\mu_0 I_c}{l} = \frac{\mu_0}{2\pi r} \left(\frac{2}{3} \frac{\mathcal{E}}{R} \right) = \frac{2 \mu_0 \mathcal{E}}{(3R)(2\pi r)}$$

$$I_c = \frac{\mathcal{E}}{R} - \frac{3\mathcal{E}}{R} = -\frac{2}{3} \frac{\mathcal{E}}{R}$$

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