Question 6

10 points total | **Distribution of points**

(a) 1 point

For a correct expression for kinetic energy — 1 point

$K = mv^2/2$

Note: This point was only awarded if no extraneous energy formulas were used.

(b) 2 points

For using the correct expression for de Broglie wavelength — 1 point

$\lambda = h/p$

For the correct answer in terms of the given quantities — 1 point

$\lambda = h/mv$

(c) 2 points

For a correct expression for the total energy of the electron and positron — 1 point

$E_{total} = 2\left(mv^2/2 + mc^2\right)$

Can also add that since $v \ll c$, $E_{total} \approx 2mc^2$

The two photons share this energy equally.

For the correct answer — 1 point

$E_{photon} = mv^2/2 + mc^2$ OR $E_{photon} \approx mc^2$

(d) 3 points

For using the given expression for the photon energy — 1 point

$E_{photon} = hf$

For expressing the energy in terms of the wavelength — 1 point

$f = c/\lambda$ so $E_{photon} = hc/\lambda$

Substituting the energy obtained in part (c)

$mv^2/2 + mc^2 = hc/\lambda$ OR $mc^2 = hc/\lambda$

For the correct answer — 1 point

$\lambda = 2hc/\left(mv^2 + 2mc^2\right)$ OR $\lambda = h/mc$

(e) 2 points

For any indication that conservation of momentum applies — 1 point

For a correct explanation of why conservation of momentum requires two photons — 1 point

Example: since the total momentum of the electron and positron was zero, the total momentum of the products must be zero. Since a photon cannot have zero momentum, two photons traveling in opposite directions are required.

Note: Only 1 point total was awarded for attempts to explain using Newton's third law.

6. (10 points)

An electron of mass m is initially moving with a constant speed v, where $v << c$. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the kinetic energy of the electron.

$K = \frac{1}{2} m v^2$.

(b) Determine the de Broglie wavelength of the electron.

$\lambda = \frac{h}{p} = \frac{h}{mv}$.

The electron encounters a particle with the same mass and opposite charge (a positron) moving with the same speed in the opposite direction. The two particles undergo a head-on collision, which results in the disappearance of both particles and the production of two photons of the same energy.

(c) Determine the energy of each photon.

$E_i = 2 \cdot m c^2$

$E_f = 2E = 2 \cdot m c^2$.

$\therefore E = mc^2$

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(d) Determine the wavelength of each photon.

$$\lambda = \frac{h}{p} = \frac{hc}{E} \quad (\because E = pc)$$

$$\Rightarrow \lambda = \frac{hc}{mc^2} = \frac{h}{mc}.$$

(e) Explain why there must be two photons produced instead of just one.

Conservation of momentum

$\Rightarrow$ Initial momentum $= 0$.

$\Rightarrow$ Final momentum must be zero.

Then, if there is only one moving photon, it doesn't satisfy the conservation of momentum.

Thus, two photons moving opossite direction must be produced.

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6. (10 points)

An electron of mass m is initially moving with a constant speed v, where $v << c$. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the kinetic energy of the electron.

$K = \frac{1}{2} m v^2$

$\boxed{K = \frac{1}{2} m_e v^2}$

(b) Determine the de Broglie wavelength of the electron.

$\lambda = \frac{h}{p}$ $p = mv$

$\lambda = \frac{h}{mv}$

$\boxed{\lambda = \frac{6.63 \times 10^{-34} J \cdot s}{m_e v}}$

The electron encounters a particle with the same mass and opposite charge (a positron) moving with the same speed in the opposite direction. The two particles undergo a head-on collision, which results in the disappearance of both particles and the production of two photons of the same energy.

(c) Determine the energy of each photon.

$E = mc^2$

$\boxed{E = 2m_e (3.0 \times 10^8 m/s)^2}$

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(d) Determine the wavelength of each photon.

$c = f\lambda$ $E = hf$ $2m_e(3.0 \times 10^8 m/s)^2 = hf$

$\lambda = \frac{c}{f}$ $\frac{2m_e(3.0 \times 10^8 m/s)}{6.63 \times 10^{-34} J \cdot s} = f$

$\lambda = \frac{(3.0 \times 10^8 m/s)}{\frac{2m_e(3.0 \times 10^8 m/s)^2}{6.63 \times 10^{-34} J \cdot s}} = \frac{\cancel{3.0 \times 10^8 m/s}}{1} \cdot \frac{6.63 \times 10^{-34} J \cdot s}{2m_e(3.0 \times 10^8 m/s)^{\cancel{2}}}$

$\boxed{\frac{6.63 \times 10^{-34} J \cdot s}{2m_e(3.0 \times 10^8 m/s)} = \lambda}$

(e) Explain why there must be two photons produced instead of just one.

There must be two photons because the energy given of by the ~~two~~ collision of the electron and positron is equal to two photons.

6. (10 points)

An electron of mass m is initially moving with a constant speed v, where $v << c$. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the kinetic energy of the electron.

$K = \frac{1}{2} mv^2$.

(b) Determine the de Broglie wavelength of the electron.

$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{mv}$ (mm)

The electron encounters a particle with the same mass and opposite charge (a positron) moving with the same speed in the opposite direction. The two particles undergo a head-on collision, which results in the disappearance of both particles and the production of two photons of the same energy.

(c) Determine the energy of each photon.

$\cancel{\frac{1}{2} mv^2 + \frac{1}{2} m(-v)^2}$

$m v_m + M v_M = m v_m' + M v_M'$

$mv + m(-v) = 0$

$\therefore$ the velocity of both particles are 0.

$E = fh = pc = mvc$

since $v = 0$ for both protons,

$\therefore E_1 = E_2 = 0$

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B6C2

(d) Determine the wavelength of each photon.

$f = \frac{v}{\lambda}$

since $v_1 = v_2 > 0$

$\therefore f_1 = f_2 = 0$.

(e) Explain why there must be two photons produced instead of just one.

The energy required to combine the two into one is:

$E = mc^2$.

However, since $v << c$, this energy cannot be reached, so the collided particles remain separate.

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Question 6

Sample: B6A
Score: 10

In part (d) the student begins with the equation $\lambda = h/p$ instead of a relationship involving energy and then makes correct substitutions to obtain the final answer.

Sample: B6B
Score: 6

This student earned full credit for parts (a) and (b) but no credit for part (c), where the energy of the particles is not correct. Part (d) earned full credit for correct work using the incorrect answer form (c). In part (e) the student tries to use an energy argument instead of momentum and received no credit.

Sample: B6C
Score: 3

Full credit was earned for parts (a) and (b), but no other credit was received. The student appears to be discussing massive particles (using the word "protons" near the end of part (c)) instead of photons.