



**AP[®] Calculus BC
2004 Sample Student Responses
Form B**

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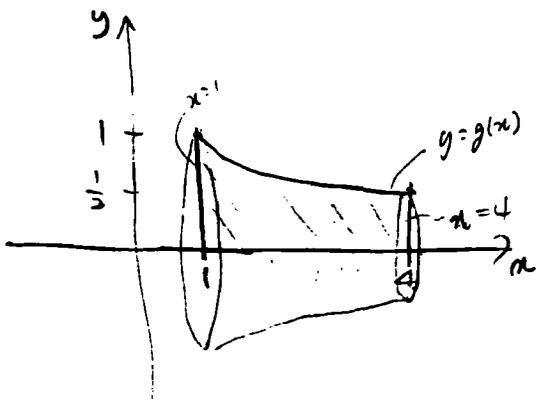
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NO CALCULATOR ALLOWED

Work for problem 5(a)

average value of g : $\frac{\int_1^4 g(x) dx}{3} = \frac{\int_1^4 x^{-\frac{1}{2}} dx}{3} = \frac{[2x^{\frac{1}{2}}]_1^4}{3} = \frac{4-2}{3} = \boxed{\frac{2}{3}}$

Work for problem 5(b)



$$\int_1^4 (g(x))^2 \cdot \pi dx$$

$$= \int_1^4 \frac{1}{x} \cdot \pi dx$$

$$= [\pi \ln x]_1^4 = \pi \ln 4 - \pi \ln 1 = \boxed{\pi \ln 4}$$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\text{Average value of the areas} = \frac{\text{Volume for } x=1 \text{ to } x=4}{3}$$

$$= \boxed{\frac{\pi \ln 4}{3}}$$

Work for problem 5(d)

$$\int_4^{\infty} g(x) dx = \lim_{n \rightarrow \infty} \int_4^n \frac{1}{\sqrt{x}} dx = \lim_{n \rightarrow \infty} \int_4^n x^{-\frac{1}{2}} dx = \lim_{n \rightarrow \infty} \left[2x^{\frac{1}{2}} \right]_4^n$$

$$= \lim_{n \rightarrow \infty} 2n^{\frac{1}{2}} - 4 = \frac{\infty}{\infty} \rightarrow \text{divergent}$$

Average value of g on the interval $[4, \infty)$

$$: \lim_{b \rightarrow \infty} \left[\frac{\int_4^b g(x) dx}{b-4} \right] = \lim_{b \rightarrow \infty} \frac{2b^{\frac{1}{2}} - 4}{b-4} = \lim_{b \rightarrow \infty} \frac{2 \cdot \frac{1}{2} b^{-\frac{1}{2}}}{1} = \lim_{b \rightarrow \infty} \frac{1}{\sqrt{b}} = \frac{0}{\infty} \rightarrow \text{finite}$$

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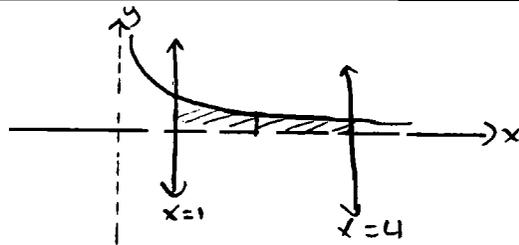
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NO CALCULATOR ALLOWED

Work for problem 5(a)

$$\begin{aligned} \text{Avg } g(x) &= \frac{1}{4-1} \int_1^4 \frac{1}{\sqrt{x}} dx = \frac{1}{3} \int_1^4 x^{-1/2} dx = \frac{1}{3} [2\sqrt{x}]_1^4 = \frac{1}{3} [2\sqrt{4} - 2\sqrt{1}] \\ &= \frac{1}{3} [2(2) - 2] = \frac{2}{3} \end{aligned}$$

Work for problem 5(b)



$$\begin{aligned} V &= \pi \int_1^4 R^2(x) - r^2(x) dx \\ &= \pi \int_1^4 \frac{1}{x} dx \\ &= \pi [\ln x]_1^4 = \pi \ln 4 \text{ units}^3 \end{aligned}$$

$$\begin{aligned} R(x) &= \frac{1}{\sqrt{x}} \Rightarrow R^2(x) = \frac{1}{x} \\ r(x) &= 0 \end{aligned}$$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

length of cross sections perpendicular to

$$x\text{-axis} = \frac{1}{\sqrt{x}} \quad A = \left(\frac{1}{\sqrt{x}}\right)^2 = \frac{1}{x}$$

$$\text{avg} = \frac{1}{4-1} \int_1^4 \left(\frac{1}{\sqrt{x}}\right)^2 dx = \frac{1}{3} \int_1^4 \frac{1}{x} dx = \frac{1}{3} [\ln x]_1^4 = \frac{1}{3} \ln 4$$

Work for problem 5(d)

$$\int_4^{\infty} g(x) dx = \lim_{a \rightarrow \infty} \int_4^a \frac{1}{\sqrt{x}} dx = \lim_{a \rightarrow \infty} [2\sqrt{x}]_4^a = \lim_{a \rightarrow \infty} 2\sqrt{a} - 2\sqrt{4} = \infty$$

\Rightarrow the improper integral $\int_4^{\infty} g(x) dx$ diverges.

$$\begin{aligned} \lim_{b \rightarrow \infty} \frac{\int_a^b f(x) dx}{b-a} &\Rightarrow \text{average value of } g = \lim_{b \rightarrow \infty} \frac{\int_a^b g(x) dx}{b-a} \\ &= \lim_{b \rightarrow \infty} \frac{\int_4^b g(x) dx}{b-4} \\ &= \lim_{b \rightarrow \infty} g(x) = 0 \end{aligned}$$

\Rightarrow avg value of g on $[4, \infty)$ is finite.

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