



**AP<sup>®</sup> Calculus BC  
2004 Scoring Guidelines  
Form B**

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**Question 1**

A particle moving along a curve in the plane has position  $(x(t), y(t))$  at time  $t$ , where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \quad \text{and} \quad \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of  $t$ . At time  $t = 0$ , the particle is at the point  $(4, 1)$ .

- (a) Find the speed of the particle and its acceleration vector at time  $t = 0$ .  
 (b) Find an equation of the line tangent to the path of the particle at time  $t = 0$ .  
 (c) Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 3$ .  
 (d) Find the  $x$ -coordinate of the position of the particle at time  $t = 3$ .

- (a) At time  $t = 0$ :

$$\text{Speed} = \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$$

$$\text{Acceleration vector} = \langle x''(0), y''(0) \rangle = \langle 0, -3 \rangle$$

$$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration vector} \end{cases}$$

(b)  $\frac{dy}{dx} = \frac{y'(0)}{x'(0)} = \frac{7}{3}$

$$\text{Tangent line is } y = \frac{7}{3}(x - 4) + 1$$

$$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \end{cases}$$

(c) Distance =  $\int_0^3 \sqrt{(\sqrt{t^4 + 9})^2 + (2e^t + 5e^{-t})^2} dt$   
 = 45.226 or 45.227

$$3 : \begin{cases} 2 : \text{distance integral} \\ \langle -1 \rangle \text{ each integrand error} \\ \langle -1 \rangle \text{ error in limits} \\ 1 : \text{answer} \end{cases}$$

(d)  $x(3) = 4 + \int_0^3 \sqrt{t^4 + 9} dt$   
 = 17.930 or 17.931

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

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**Question 2**

Let  $f$  be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for  $f$  about  $x = 2$  is given by  $T(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3$ .

- (a) Find  $f(2)$  and  $f''(2)$ .
- (b) Is there enough information given to determine whether  $f$  has a critical point at  $x = 2$ ?  
 If not, explain why not. If so, determine whether  $f(2)$  is a relative maximum, a relative minimum, or neither, and justify your answer.
- (c) Use  $T(x)$  to find an approximation for  $f(0)$ . Is there enough information given to determine whether  $f$  has a critical point at  $x = 0$ ? If not, explain why not. If so, determine whether  $f(0)$  is a relative maximum, a relative minimum, or neither, and justify your answer.
- (d) The fourth derivative of  $f$  satisfies the inequality  $|f^{(4)}(x)| \leq 6$  for all  $x$  in the closed interval  $[0, 2]$ . Use the Lagrange error bound on the approximation to  $f(0)$  found in part (c) to explain why  $f(0)$  is negative.

(a)  $f(2) = T(2) = 7$   
 $\frac{f''(2)}{2!} = -9$  so  $f''(2) = -18$

2 :  $\begin{cases} 1 : f(2) = 7 \\ 1 : f''(2) = -18 \end{cases}$

(b) Yes, since  $f'(2) = T'(2) = 0$ ,  $f$  does have a critical point at  $x = 2$ .  
 Since  $f''(2) = -18 < 0$ ,  $f(2)$  is a relative maximum value.

2 :  $\begin{cases} 1 : \text{states } f'(2) = 0 \\ 1 : \text{declares } f(2) \text{ as a relative maximum because } f''(2) < 0 \end{cases}$

(c)  $f(0) \approx T(0) = -5$   
 It is not possible to determine if  $f$  has a critical point at  $x = 0$  because  $T(x)$  gives exact information only at  $x = 2$ .

3 :  $\begin{cases} 1 : f(0) \approx T(0) = -5 \\ 1 : \text{declares that it is not possible to determine} \\ 1 : \text{reason} \end{cases}$

(d) Lagrange error bound  $= \frac{6}{4!}|0 - 2|^4 = 4$   
 $f(0) \leq T(0) + 4 = -1$   
 Therefore,  $f(0)$  is negative.

2 :  $\begin{cases} 1 : \text{value of Lagrange error bound} \\ 1 : \text{explanation} \end{cases}$

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**Question 3**

A test plane flies in a straight line with positive velocity  $v(t)$ , in miles per minute at time  $t$  minutes, where  $v$  is a differentiable function of  $t$ . Selected values of  $v(t)$  for  $0 \leq t \leq 40$  are shown in the table above.

$t$ (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate  $\int_0^{40} v(t) dt$ . Show the computations that lead to your answer. Using correct units, explain the meaning of  $\int_0^{40} v(t) dt$  in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval  $0 < t < 40$ ? Justify your answer.
- (c) The function  $f$ , defined by  $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$ , is used to model the velocity of the plane, in miles per minute, for  $0 \leq t \leq 40$ . According to this model, what is the acceleration of the plane at  $t = 23$ ? Indicate units of measure.
- (d) According to the model  $f$ , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval  $0 \leq t \leq 40$ ?

(a) Midpoint Riemann sum is  
 $10 \cdot [v(5) + v(15) + v(25) + v(35)]$   
 $= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$   
 The integral gives the total distance in miles that the plane flies during the 40 minutes.

3 :  $\left\{ \begin{array}{l} 1 : v(5) + v(15) + v(25) + v(35) \\ 1 : \text{answer} \\ 1 : \text{meaning with units} \end{array} \right.$

(b) By the Mean Value Theorem,  $v'(t) = 0$  somewhere in the interval  $(0, 15)$  and somewhere in the interval  $(25, 30)$ . Therefore the acceleration will equal 0 for at least two values of  $t$ .

2 :  $\left\{ \begin{array}{l} 1 : \text{two instances} \\ 1 : \text{justification} \end{array} \right.$

(c)  $f'(23) = -0.407$  or  $-0.408$  miles per minute<sup>2</sup>

1 : answer with units

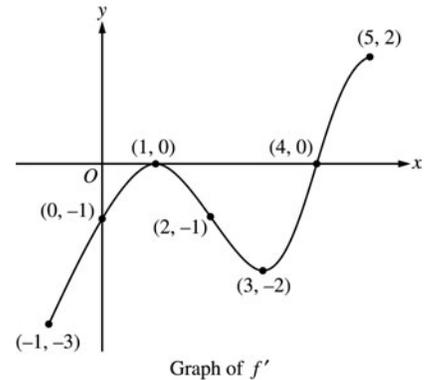
(d) Average velocity  $= \frac{1}{40} \int_0^{40} f(t) dt$   
 $= 5.916$  miles per minute

3 :  $\left\{ \begin{array}{l} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

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**Question 4**

The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , on the closed interval  $-1 \leq x \leq 5$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$  and  $x = 3$ . The function  $f$  is twice differentiable with  $f(2) = 6$ .



- (a) Find the  $x$ -coordinate of each of the points of inflection of the graph of  $f$ . Give a reason for your answer.
- (b) At what value of  $x$  does  $f$  attain its absolute minimum value on the closed interval  $-1 \leq x \leq 5$ ? At what value of  $x$  does  $f$  attain its absolute maximum value on the closed interval  $-1 \leq x \leq 5$ ? Show the analysis that leads to your answers.
- (c) Let  $g$  be the function defined by  $g(x) = xf(x)$ . Find an equation for the line tangent to the graph of  $g$  at  $x = 2$ .

- (a)  $x = 1$  and  $x = 3$  because the graph of  $f'$  changes from increasing to decreasing at  $x = 1$ , and changes from decreasing to increasing at  $x = 3$ .

$$2 : \begin{cases} 1 : x = 1, x = 3 \\ 1 : \text{reason} \end{cases}$$

- (b) The function  $f$  decreases from  $x = -1$  to  $x = 4$ , then increases from  $x = 4$  to  $x = 5$ . Therefore, the absolute minimum value for  $f$  is at  $x = 4$ . The absolute maximum value must occur at  $x = -1$  or at  $x = 5$ .

$$4 : \begin{cases} 1 : \text{indicates } f \text{ decreases then increases} \\ 1 : \text{eliminates } x = 5 \text{ for maximum} \\ 1 : \text{absolute minimum at } x = 4 \\ 1 : \text{absolute maximum at } x = -1 \end{cases}$$

$$f(5) - f(-1) = \int_{-1}^5 f'(t) dt < 0$$

Since  $f(5) < f(-1)$ , the absolute maximum value occurs at  $x = -1$ .

- (c)  $g'(x) = f(x) + xf'(x)$   
 $g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$   
 $g(2) = 2f(2) = 12$

$$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$$

Tangent line is  $y = 4(x - 2) + 12$

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**Question 5**

Let  $g$  be the function given by  $g(x) = \frac{1}{\sqrt{x}}$ .

- (a) Find the average value of  $g$  on the closed interval  $[1, 4]$ .
- (b) Let  $S$  be the solid generated when the region bounded by the graph of  $y = g(x)$ , the vertical lines  $x = 1$  and  $x = 4$ , and the  $x$ -axis is revolved about the  $x$ -axis. Find the volume of  $S$ .
- (c) For the solid  $S$ , given in part (b), find the average value of the areas of the cross sections perpendicular to the  $x$ -axis.
- (d) The average value of a function  $f$  on the unbounded interval  $[a, \infty)$  is defined to be

$\lim_{b \rightarrow \infty} \left[ \frac{\int_a^b f(x) dx}{b - a} \right]$ . Show that the improper integral  $\int_4^{\infty} g(x) dx$  is divergent, but the average value of  $g$  on the interval  $[4, \infty)$  is finite.

(a)  $\frac{1}{3} \int_1^4 \frac{1}{\sqrt{x}} dx = \frac{1}{3} \cdot 2\sqrt{x} \Big|_1^4 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

2 :  $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{antidifferentiation} \\ \text{and evaluation} \end{array} \right.$

(b) Volume =  $\pi \int_1^4 \frac{1}{x} dx = \pi \ln x \Big|_1^4 = \pi \ln 4$

2 :  $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{antidifferentiation} \\ \text{and evaluation} \end{array} \right.$

(c) The cross section at  $x$  has area  $\pi \left( \frac{1}{\sqrt{x}} \right)^2 = \frac{\pi}{x}$

1 : answer

Average value =  $\frac{1}{3} \int_1^4 \frac{\pi}{x} dx = \frac{1}{3} \pi \ln 4$

(d)  $\int_4^{\infty} g(x) dx = \lim_{b \rightarrow \infty} \int_4^b \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow \infty} (2\sqrt{b} - 4) = \infty$

This limit is not finite, so the integral is divergent.

$$\frac{\int_4^b g(x) dx}{b - 4} = \frac{1}{b - 4} \int_4^b \frac{1}{\sqrt{x}} dx = \frac{2\sqrt{b} - 4}{b - 4}$$

$$\lim_{b \rightarrow \infty} \frac{2\sqrt{b} - 4}{b - 4} = 0$$

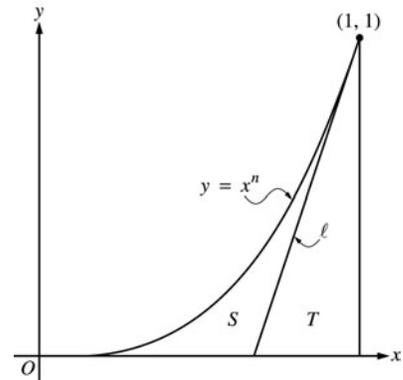
4 :  $\left\{ \begin{array}{l} 1 : \int_4^b g(x) dx = 2\sqrt{b} - 4 \\ 1 : \text{indicates integral diverges} \\ 1 : \frac{1}{b - 4} \int_4^b g(x) dx = \frac{2\sqrt{b} - 4}{b - 4} \\ 1 : \text{finite limit as } b \rightarrow \infty \end{array} \right.$

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**Question 6**

Let  $\ell$  be the line tangent to the graph of  $y = x^n$  at the point  $(1, 1)$ , where  $n > 1$ , as shown above.

- (a) Find  $\int_0^1 x^n dx$  in terms of  $n$ .
- (b) Let  $T$  be the triangular region bounded by  $\ell$ , the  $x$ -axis, and the line  $x = 1$ . Show that the area of  $T$  is  $\frac{1}{2n}$ .
- (c) Let  $S$  be the region bounded by the graph of  $y = x^n$ , the line  $\ell$ , and the  $x$ -axis. Express the area of  $S$  in terms of  $n$  and determine the value of  $n$  that maximizes the area of  $S$ .



(a)  $\int_0^1 x^n dx = \frac{x^{n+1}}{n+1} \Big|_0^1 = \frac{1}{n+1}$

2 :  $\left\{ \begin{array}{l} 1 : \text{antiderivative of } x^n \\ 1 : \text{answer} \end{array} \right.$

- (b) Let  $b$  be the length of the base of triangle  $T$ .

$\frac{1}{b}$  is the slope of line  $\ell$ , which is  $n$

3 :  $\left\{ \begin{array}{l} 1 : \text{slope of line } \ell \text{ is } n \\ 1 : \text{base of } T \text{ is } \frac{1}{n} \\ 1 : \text{shows area is } \frac{1}{2n} \end{array} \right.$

$$\text{Area}(T) = \frac{1}{2}b(1) = \frac{1}{2n}$$

(c)  $\text{Area}(S) = \int_0^1 x^n dx - \text{Area}(T)$   
 $= \frac{1}{n+1} - \frac{1}{2n}$

4 :  $\left\{ \begin{array}{l} 1 : \text{area of } S \text{ in terms of } n \\ 1 : \text{derivative} \\ 1 : \text{sets derivative equal to } 0 \\ 1 : \text{solves for } n \end{array} \right.$

$$\frac{d}{dn} \text{Area}(S) = -\frac{1}{(n+1)^2} + \frac{1}{2n^2} = 0$$

$$2n^2 = (n+1)^2$$

$$\sqrt{2}n = (n+1)$$

$$n = \frac{1}{\sqrt{2}-1} = 1 + \sqrt{2}$$