



AP[®] Calculus BC 2002 Sample Student Responses

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Work for problem 6(a)

$$(a) \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2x)^{n+2}}{(n+2)} \cdot \frac{n+1}{(2x)^{n+1}} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(2x)(n+1)}{n+2} \right| = |2x| \cdot \lim_{n \rightarrow \infty} \left| \frac{n+1}{n+2} \right| < 1$$

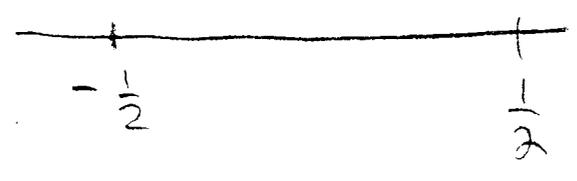
$$|2x| < 1$$

$$|2x| < 1$$

$$-1 < 2x < 1$$

$$-\frac{1}{2} < x < \frac{1}{2}$$

$$-\frac{1}{2} \leq x < \frac{1}{2}$$



$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$$

$$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \left| \frac{1}{n+1} \right| = 0 \quad (ii) \checkmark$$

$$\left| \frac{a_{n+1}}{a_n} \right| < 1$$

$$\left| \frac{\frac{1}{n+2}}{\frac{1}{n+1}} \right| = \left| \frac{n+1}{n+2} \right| < 1 \checkmark$$

converges by the alternating series test

$$\int_0^{\infty} \frac{1}{n+1} \, dn$$

$$= \lim_{b \rightarrow \infty} \int_0^b \frac{1}{n+1} \, dn$$

$$= \lim_{b \rightarrow \infty} \ln|n+1| \Big|_0^b$$

$$= \lim_{b \rightarrow \infty} \ln|b+1| - \ln|1|$$

$$= \infty - 0$$

divergent by the integral test

Work for problem 6(b)

$$f'(x) \approx \overset{n=0}{2} + \overset{n=1}{\frac{8x}{2}} + \overset{n=2}{\frac{24x^2}{3}} + \overset{n=3}{\frac{64x^3}{4}} + \dots$$

$$f'(x) \approx \boxed{2 + 4x + 8x^2 + 16x^3 + \dots + 2^{(n+1)}x^n}$$

$$\begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array}$$

$$2^{n+1} \quad 2^2 \quad 2^3 \quad 2^4$$

Work for problem 6(c)

$$f'(-\frac{1}{3}) \approx 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \dots + 2^{n+1} \left(-\frac{1}{3}\right)^n$$

$$\sum_{n=0}^{\infty} 2^{n+1} \left(-\frac{1}{3}\right)^n = \sum_{n=0}^{\infty} \frac{2^n \cdot 2 \cdot (-1)^n}{3^n} = \sum_{n=0}^{\infty} 2 \cdot \left(-\frac{2}{3}\right)^n$$

$$a = 2$$

$$r = -\frac{2}{3}$$

$$S = \frac{a}{1-r} = \frac{2}{1 - (-\frac{2}{3})} = \frac{2}{\frac{5}{3}} \cdot \frac{3}{3} = \boxed{\frac{6}{5}}$$

6

6

6

6

6

6

6

6

6

6

NO CALCULATOR ALLOWED

C₁

Work for problem 6(a)

$$\frac{(2x)^{n+1}}{n+1}$$

$$\lim_{n \rightarrow \infty} \frac{(2x)^{(n+1)+1}}{(n+1)+1} \cdot \frac{n+1}{(2x)^{n+1}}$$

$$\lim_{n \rightarrow \infty} \frac{(2x)^{n+2}}{n+2} \cdot \frac{n+1}{(2x)^{n+1}}$$

$$\lim_{n \rightarrow \infty} \frac{(2x)^{n+1}}{n+2}$$

$$-1 < 2x < 1$$

$$-\frac{1}{2} < x < \frac{1}{2}$$

$$\sum_{n=0}^{\infty} \frac{2\left(\frac{1}{2}\right)^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty} \frac{(1)^{n+1}}{n+1}$$

converges
when $x = \frac{1}{2}$

$$\sum_{n=0}^{\infty} \frac{2\left(-\frac{1}{2}\right)^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$$

converges
when $x = -\frac{1}{2}$

① ~~terms~~ decrease② ~~term~~ $\rightarrow 0$

③ signs alternate

$$-\frac{1}{2} \leq x \leq \frac{1}{2}$$

6

6

6

6

6

6

6

6

6

6

NO CALCULATOR ALLOWED

C₂

Work for problem 6(b)

$$f(x) \approx 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4}$$

$$f'(x) \approx 2 + 4x + 8x^2 + 16x^3$$

$$f'(x) = 2 + 4x + 8x^2 + 16x^3 + \dots + 2^{n+1}x^n + \dots$$

Work for problem 6(c)

$$f(x) = 2 + \frac{4(2)x}{2} + \frac{8(2)x^2}{3} + \frac{16(4)x^3}{4}$$

$$f'\left(\frac{1}{3}\right) = 2 + 4\left(\frac{1}{3}\right) + 8\left(\frac{1}{9}\right) + 16\left(\frac{1}{27}\right)$$

$$\sum_{n=0}^{\infty} 2^{n+1} x^n$$

$$\sum_{n=0}^{\infty} \frac{2^{n+1} \left(\frac{1}{3}\right)^n}{}$$