



AP[®] Calculus BC
2006 Free-Response Questions
Form B

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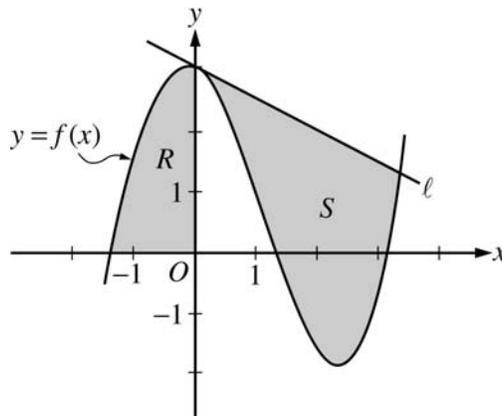
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2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f , and let S be the shaded region bounded by the graph of f and line ℓ , the line tangent to the graph of f at $x = 0$, as shown above.
- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- (c) Write, but do not evaluate, an integral expression that can be used to find the area of S .

WRITE ALL WORK IN THE EXAM BOOKLET.

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2. An object moving along a curve in the xy -plane is at position $(x(t), y(t))$ at time t , where

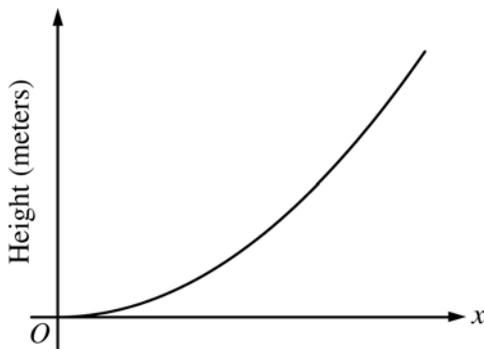
$$\frac{dx}{dt} = \tan(e^{-t}) \text{ and } \frac{dy}{dt} = \sec(e^{-t})$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

- (a) Write an equation for the line tangent to the curve at position $(2, -3)$.
 - (b) Find the acceleration vector and the speed of the object at time $t = 1$.
 - (c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.
 - (d) Is there a time $t \geq 0$ at which the object is on the y -axis? Explain why or why not.
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3. The figure above is the graph of a function of x , which models the height of a skateboard ramp. The function meets the following requirements.
- (i) At $x = 0$, the value of the function is 0, and the slope of the graph of the function is 0.
 - (ii) At $x = 4$, the value of the function is 1, and the slope of the graph of the function is 1.
 - (iii) Between $x = 0$ and $x = 4$, the function is increasing.
- (a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.
- (b) Let $g(x) = cx^3 - \frac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirement (ii) above. Show the work that leads to your answer.
- (c) Using the function g and your value of c from part (b), show that g does not meet requirement (iii) above.
- (d) Let $h(x) = \frac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirement (ii) above. Show that h also meets requirements (i) and (iii) above.
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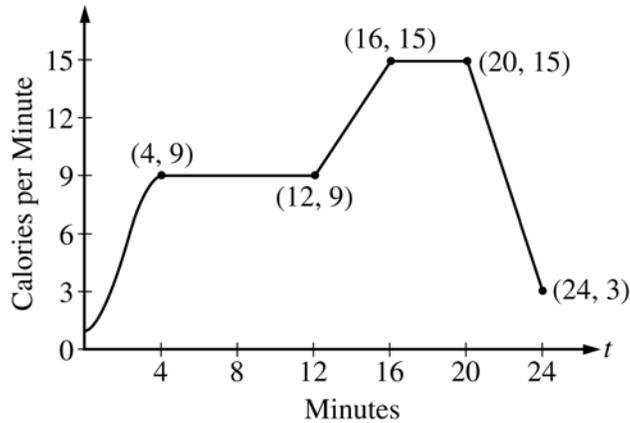
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END OF PART A OF SECTION II

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CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f . In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \leq t \leq 4$ and f is piecewise linear for $4 \leq t \leq 24$.
- (a) Find $f'(22)$. Indicate units of measure.
- (b) For the time interval $0 \leq t \leq 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.
- (c) Find the total number of calories burned over the time interval $6 \leq t \leq 18$ minutes.
- (d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \leq t \leq 18$.

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5. Let f be a function with $f(4) = 1$ such that all points (x, y) on the graph of f satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x).$$

Let g be a function with $g(4) = 1$ such that all points (x, y) on the graph of g satisfy the logistic differential equation

$$\frac{dy}{dx} = 2y(3 - y).$$

- (a) Find $y = f(x)$.
- (b) Given that $g(4) = 1$, find $\lim_{x \rightarrow \infty} g(x)$ and $\lim_{x \rightarrow \infty} g'(x)$. (It is not necessary to solve for $g(x)$ or to show how you arrived at your answers.)
- (c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for $g(x)$.)
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6. The function f is defined by $f(x) = \frac{1}{1 + x^3}$. The Maclaurin series for f is given by

$$1 - x^3 + x^6 - x^9 + \cdots + (-1)^n x^{3n} + \cdots,$$

which converges to $f(x)$ for $-1 < x < 1$.

- (a) Find the first three nonzero terms and the general term for the Maclaurin series for $f'(x)$.
- (b) Use your results from part (a) to find the sum of the infinite series $-\frac{3}{2^2} + \frac{6}{2^5} - \frac{9}{2^8} + \cdots + (-1)^n \frac{3n}{2^{3n-1}} + \cdots$.
- (c) Find the first four nonzero terms and the general term for the Maclaurin series representing $\int_0^x f(t) dt$.
- (d) Use the first three nonzero terms of the infinite series found in part (c) to approximate $\int_0^{1/2} f(t) dt$. What are the properties of the terms of the series representing $\int_0^{1/2} f(t) dt$ that guarantee that this approximation is within $\frac{1}{10,000}$ of the exact value of the integral?
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END OF EXAM

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