



AP[®] Calculus BC 2011 Free-Response Questions

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2011 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

**CALCULUS BC
SECTION II, Part A**

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

1. At time t , a particle moving in the xy -plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.
- (a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.
 - (b) Find the slope of the line tangent to the path of the particle at time $t = 3$.
 - (c) Find the position of the particle at time $t = 3$.
 - (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

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t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.
- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

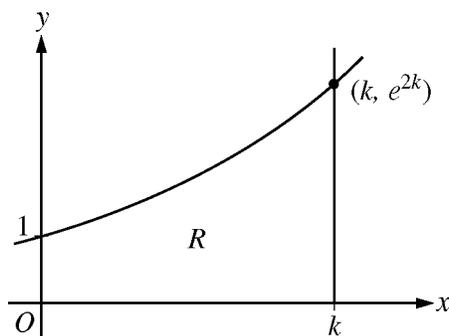
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CALCULUS BC
SECTION II, Part B

Time—60 minutes

Number of problems—4

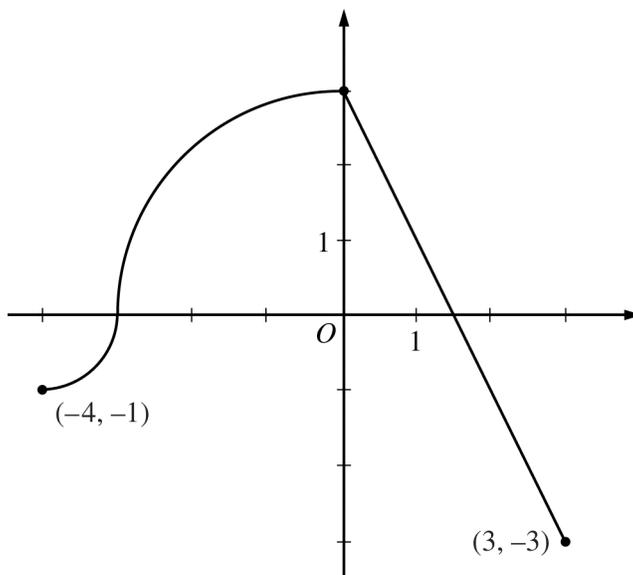
No calculator is allowed for these problems.



3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f , the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.
- (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k .
- (b) The region R is rotated about the x -axis to form a solid. Find the volume, V , of the solid in terms of k .
- (c) The volume V , found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

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Graph of f

4. The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t) dt$.
- Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
 - Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.
 - Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
 - Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

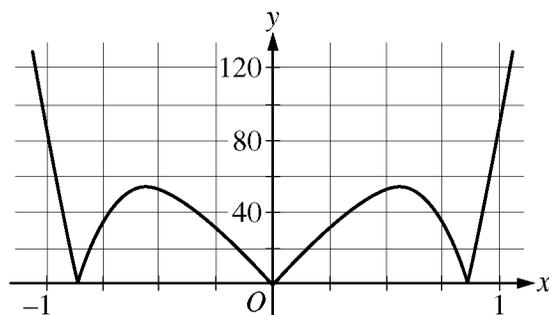
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5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.
- (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).
- (b) Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.
- (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.
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Graph of $y = |f^{(5)}(x)|$

6. Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = |f^{(5)}(x)|$ is shown above.
- Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about $x = 0$.
 - Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin(x^2)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x = 0$.
 - Find the value of $f^{(6)}(0)$.
 - Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

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END OF EXAM