



## AP<sup>®</sup> Calculus BC 2002 Sample Student Responses Form B

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NO CALCULATOR ALLOWED

Work for problem 5(a)

1) Solving the equation

$$\frac{dy}{dx} = \frac{3-x}{y}$$

$$y dy = (3-x) dx$$

$$\int y dy = \int (3-x) dx$$

$$\frac{y^2}{2} = 3x - \frac{x^2}{2} + C$$

$$y^2 = 6x - x^2 + C \text{ General solution.}$$

2) Because  $y = -2$  is tangent to  $f(x)$  at  $(x_0, -2)$ ,

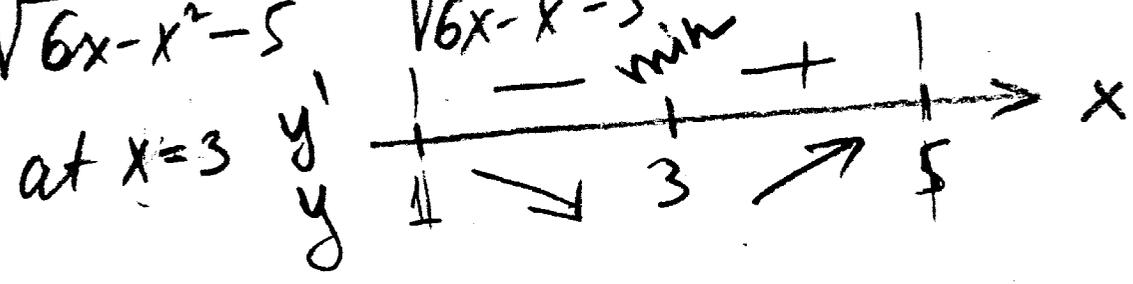
$$\left. \frac{dy}{dx} \right|_{\substack{x=x_0 \\ y=-2}} = 0$$

$$\frac{3-x_0}{-2} = 0$$
  
$$x_0 = 3$$

3) Particular solution  $y = -\sqrt{6x}$

$$(3, -2) \quad 4 = 6 \cdot 3 - 9 + C$$
  
$$C = -5$$

$$4) \quad y' = \frac{-(3-x)}{\sqrt{6x-x^2-5}} = \frac{x-3}{\sqrt{6x-x^2-5}}$$



at  $x = 3$   $y = f(x)$  has a local minimum.

NO CALCULATOR ALLOWED

Work for problem 5(b)

$$\frac{dy}{dx} = \frac{3-x}{y}$$

$$y^2 = 6x - x^2 + C \text{ as found in a(1).}$$

$$y(6) = -4 \quad 16 = 6 \cdot (6) - 36 + C$$
$$C = 16$$

$$y^2 = 6x - x^2 + 16$$

$$y = -\sqrt{6x - x^2 + 16}$$

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NO CALCULATOR ALLOWED

Work for problem 5(a)

1 < x < 5

$$y' = \frac{3-x}{y}$$

$$\frac{dy}{dx} = \frac{3-x}{y}$$

$$0 = \frac{3-x}{y}$$

$$0 = 3-x$$

~~Point of tangency at x=3~~

3 = x - coordinate of point of tangency, local maximum

$$y = \pm \sqrt{6x - x^2}$$

$$x=0 \\ y=0$$

$$x=1 \quad y=\sqrt{5}$$

$$x=4 \quad y=2 \quad \text{be}$$

$$\frac{2}{\sqrt{5}} \text{ inc}$$

$$-\frac{1}{8} \text{ c}$$

I	f'(x)
$-\infty, 3$	inc
$(3, \infty)$	dec

$$y = \pm \sqrt{6-x}$$

NO CALCULATOR ALLOWED

Work for problem 5(b)

(6, -4)

~~$$\int y dy = \int 3-x dx$$

$$\frac{1}{2}y^2 =$$~~

$$\int y dy = \int 3-x dx$$

$$\frac{1}{2}y^2 + C = 3x - \frac{1}{2}x^2 + C$$

$$y = \sqrt{6x - x^2 + C}$$

$$-4 = \sqrt{36 - 36 + C}$$

$$-4 = \pm \sqrt{C}$$

$$g(x) = -\sqrt{6x - x^2 + 16}$$

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