



AP[®] Calculus BC 2003 Sample Student Responses Form B

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Work for problem 6(a)

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$\Rightarrow f(x) = 1 + \frac{2}{3}(x-2) + \frac{3}{9}(x-2)^2 + \frac{4}{27}(x-2)^3 + \dots + \frac{n+1}{3^n}(x-2)^n + \dots$$

Work for problem 6(b)

Using the ratio test:

$$\begin{aligned} L &= \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{n+2}{3^{n+1}}(x-2)^{n+1}}{\frac{n+1}{3^n}(x-2)^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{n+2}{n+1} \cdot \frac{x-2}{3} \right| = \frac{x-2}{3} \end{aligned}$$

\Rightarrow since $|L| < 1$ for series to converge:

$$-1 < \frac{x-2}{3} < 1 \quad \Rightarrow \quad -1 < x < 5$$

\Rightarrow The radius of convergence is 3.

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$g(2) = 3, \quad g^{(n)}(2) = \frac{n!}{3^{n-1}}$$

$$\Rightarrow g(x) = 3 + (x-2) + \frac{1}{2}(x-2)^2 + \frac{1}{6}(x-2)^3 + \dots + \frac{1}{3^{n-1}}(x-2)^n + \dots$$

Work for problem 6(d)

Since $T_n = \frac{(-4)^n}{3^{n-1}}$ if $x = -2$, Then,

$$T_n = (-1)^n \cdot 3 \left(\frac{4}{3}\right)^n$$

\Rightarrow alternating geometric sequence with $r = \frac{4}{3} > 1$; it is neither absolutely convergent or conditionally convergent, hence g does not converge at $x = -2$.

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.

Work for problem 6(a)

$$f^{(n)}(2) = \frac{(n+1)!}{3^n}$$

$$f(x) = 1 + \frac{2!}{3}(x-2) + \frac{3!}{2!3^2}(x-2)^2 + \frac{4!}{3!3^3}(x-2)^3 + \dots + \frac{n!}{3^n}(x-2)^n$$

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Work for problem 6(b)

$$f(x) = \sum_0^{\infty} \frac{n}{3^n} (x-2)^n$$

by ratio test, $\lim_{n \rightarrow \infty} \left| \frac{(n+2) \left(\frac{x-2}{3}\right)^{n+1}}{(n+1) \left(\frac{x-2}{3}\right)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+2)}{(n+1)} \left(\frac{x-2}{3}\right) \right| \rightarrow \left| \frac{x-2}{3} \right|$

$f(x)$ converges if $\left| \frac{x-2}{3} \right| < 1$
 $|x-2| < 3$

\therefore the radius of convergence of f about $x=2$
 is $2/1$

Continue problem 6 on page 15.

Work for problem 6(c)

$g(2) = 3 ; g' = f(x)$

$g = \int f(x) dx = x + C + \frac{1}{3}(x-2)^2 + \frac{1}{3^2}(x-2)^3 + \frac{1}{3^3}(x-2)^4 + \dots + \frac{1}{3^n}(x-2)^{n+1}$

Work for problem 6(d)

by ratio test, $\lim_{n \rightarrow \infty} \left| \frac{\left(\frac{x-2}{3}\right)^{n+1}}{\left(\frac{x-2}{3}\right)^n} \right| \rightarrow \left| \frac{x-2}{3} \right|$

$\therefore g$ converges if $\left| \frac{x-2}{3} \right| < 1$
 $-1 < x < 5$

$x = -2$ is outside of the range of convergence
 $\therefore g$ is not converge at $x = -2$

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