



**AP[®] Calculus BC
2004 Free-Response Questions
Form B**

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2004 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t , where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t . At time $t = 0$, the particle is at the point $(4, 1)$.

- (a) Find the speed of the particle and its acceleration vector at time $t = 0$.
- (b) Find an equation of the line tangent to the path of the particle at time $t = 0$.
- (c) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.
- (d) Find the x -coordinate of the position of the particle at time $t = 3$.
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2. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by

$$T(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3.$$

- (a) Find $f(2)$ and $f''(2)$.
- (b) Is there enough information given to determine whether f has a critical point at $x = 2$?
If not, explain why not.
If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
- (c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether f has a critical point at $x = 0$?
If not, explain why not.
If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
- (d) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)| \leq 6$ for all x in the closed interval $[0, 2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.
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t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.
- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

END OF PART A OF SECTION II

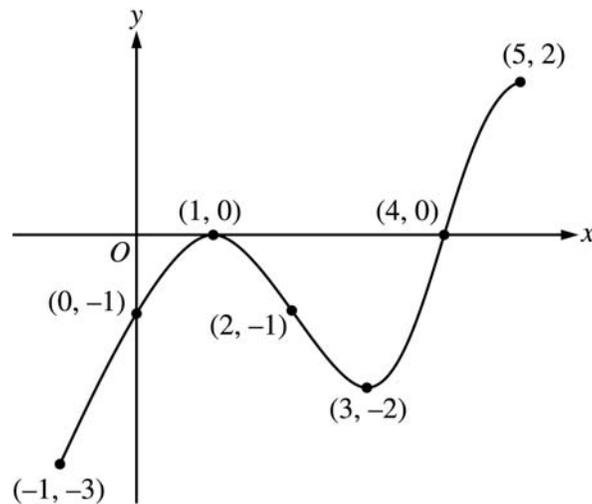
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CALCULUS BC
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



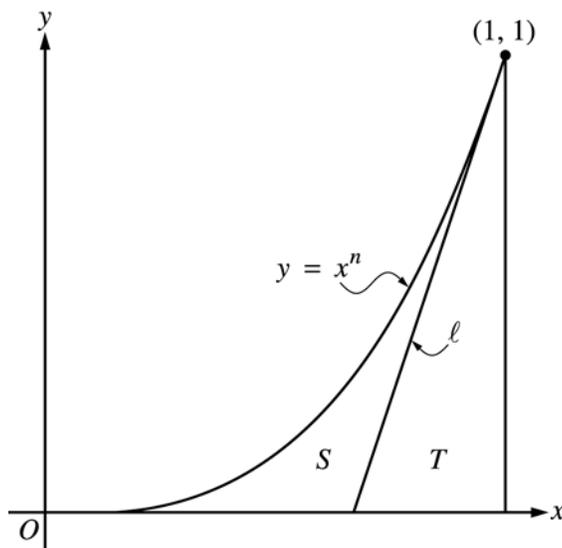
Graph of f'

4. The figure above shows the graph of f' , the derivative of the function f , on the closed interval $-1 \leq x \leq 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.
- (a) Find the x -coordinate of each of the points of inflection of the graph of f . Give a reason for your answer.
- (b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \leq x \leq 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \leq x \leq 5$? Show the analysis that leads to your answers.
- (c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

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5. Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

- (a) Find the average value of g on the closed interval $[1, 4]$.
- (b) Let S be the solid generated when the region bounded by the graph of $y = g(x)$, the vertical lines $x = 1$ and $x = 4$, and the x -axis is revolved about the x -axis. Find the volume of S .
- (c) For the solid S , given in part (b), find the average value of the areas of the cross sections perpendicular to the x -axis.
- (d) The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b \rightarrow \infty} \left[\frac{\int_a^b f(x) dx}{b - a} \right]$. Show that the improper integral $\int_4^{\infty} g(x) dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.



6. Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.
- (a) Find $\int_0^1 x^n dx$ in terms of n .
- (b) Let T be the triangular region bounded by ℓ , the x -axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.
- (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ , and the x -axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S .

END OF EXAMINATION