



## AP Calculus BC 1999 Sample Student Responses

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## CALCULUS BC

## SECTION II

Time—1 hour and 30 minutes

Number of problems—6

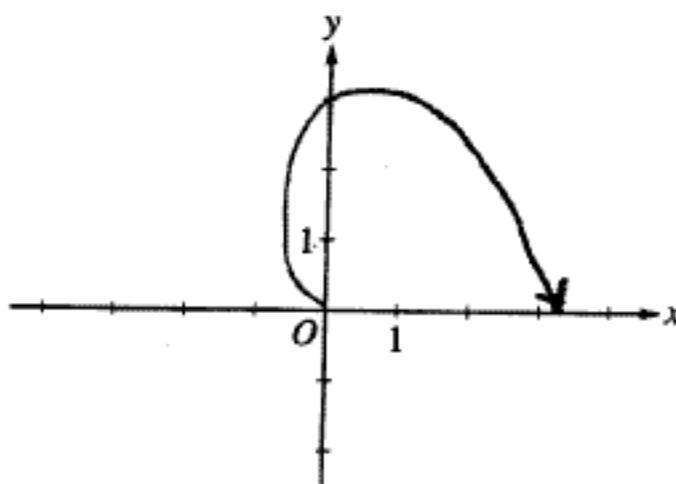
Percent of total grade—50

**REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.**

1. A particle moves in the  $xy$ -plane so that its position at any time  $t$ ,  $0 \leq t \leq \pi$ , is given by

$$x(t) = \frac{t^2}{2} - \ln(1+t) \text{ and } y(t) = 3 \sin t.$$

- (a) Sketch the path of the particle in the  $xy$ -plane below. Indicate the direction of motion along the path.



Continue problem 1 on page 5.

(b) At what time  $t$ ,  $0 \leq t \leq \pi$ , does  $x(t)$  attain its minimum value? What is the position  $(x(t), y(t))$  of the particle at this time?

$$x(t) = \frac{t^2}{2} - \ln(1+t)$$

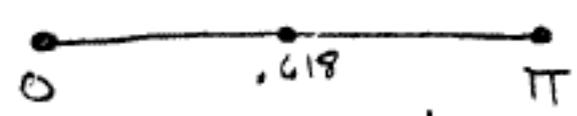
$$0 = t - \frac{1}{1+t}$$

$$x'(t) = t - \frac{1}{1+t}$$

$$t(t+1) - 1 = 0$$

$$t^2 + t - 1 = 0$$

$$t = .618$$



$x'(t)$  -            +  
 $x(t)$  dec.        inc  
 $\Rightarrow \therefore$  Minimum at  $t = .618$

$$P(.618) = \left( \frac{(.618)^2}{2} - \ln(1+.618), 3 \sin(.618) \right)$$

$$P(.618) = (-.290, 1.738)$$

(c) At what time  $t$ ,  $0 < t < \pi$ , is the particle on the  $y$ -axis? Find the speed and the acceleration vector of the particle at this time.

When  $x=0$ , the particle is on the  $y$  axis

$$\Rightarrow x(t) = 0 \Rightarrow \frac{t^2}{2} - \ln(1+t) = 0$$

$\therefore$  when  $t = 1.286$  the particle is on the  $y$ -axis

$$\text{Velocity}_x = x'(t) = t - \frac{1}{1+t}$$

$$\text{Velocity}_y = 3 \cos t$$

$$\text{Speed} = \sqrt{x'(t)^2 + y'(t)^2}$$

$$= \sqrt{\left(t - \frac{1}{1+t}\right)^2 + (3 \cos t)^2}$$

$$\text{Speed at } (t=1.286) = 1.196$$

$$\text{Acceleration} = (\text{Velocity})'$$

$$A(t) = \left( 1 + \frac{1}{(1+t)^2}, -3 \sin t \right)$$

$$A(1.286) = (1.191, -2.879)$$



## CALCULUS BC

## SECTION II

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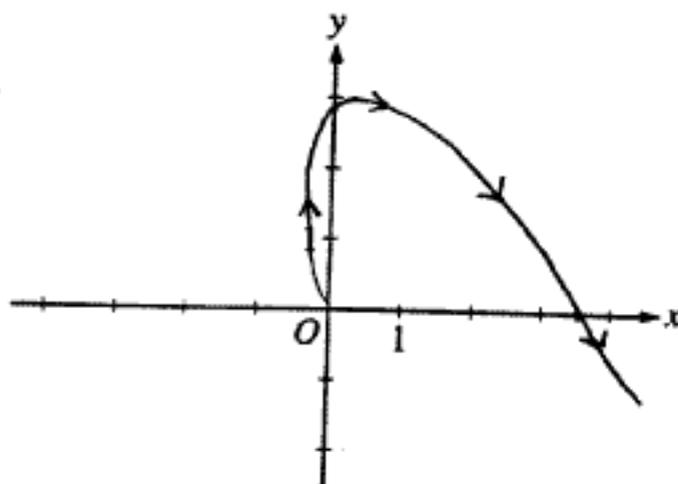
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$$x(t) = \frac{t^2}{2} - \ln(1 + t) \text{ and } y(t) = 3 \sin t.$$

- (a) Sketch the path of the particle in the  $xy$ -plane below. Indicate the direction of motion along the path.



D

- (b) At what time  $t$ ,  $0 \leq t \leq \pi$ , does  $x(t)$  attain its minimum value? What is the position  $(x(t), y(t))$  of the particle at this time?

$$x(t) = \frac{t^2}{2} - \ln(1+t)$$

$$\frac{dx}{dt} = t - \frac{1}{1+t}$$

$$0 = t - \frac{1}{1+t}$$

$$t = \frac{1}{1+t}$$

$$t^2 + t = 1$$

$$t^2 + t - 1 = 0$$

$$t = \frac{-1 + \sqrt{5}}{2} \quad \text{by quadratic formula}$$

$$x(t) = -0.290$$

$$y(t) = 1.73$$

position is  $(-0.290, 1.73)$

- (c) At what time  $t$ ,  $0 < t < \pi$ , is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

$$x(t) = \frac{t^2}{2} - \ln(1+t)$$

$$0 = \frac{t^2}{2} - \ln(1+t)$$

$$\ln(1+t) = \frac{t^2}{2}$$

$t = 0$  at 0 and 1.285

$t$  is on the y-axis at

$$1.285 \text{ for } 0 < t < \pi$$

$$\frac{dx}{dt} = t - \frac{1}{1+t} \quad \frac{dy}{dt} = 3 \cos t$$

$$v(t) = \left( t - \frac{1}{1+t}, 3 \cos t \right)$$

speed = magnitude of velocity

$$\frac{dx}{dt} = 1.848 \quad \frac{dy}{dt} = 1.843$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 1.196 = \text{speed}$$

$$v(t) = \left( t - \frac{1}{1+t}, 3 \cos t \right)$$

$$a(t) = \left( 1 + \frac{1}{(1+t)^2}, -3 \sin t \right)$$

$$a(t) = (1.191, -2.879)$$

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CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

F

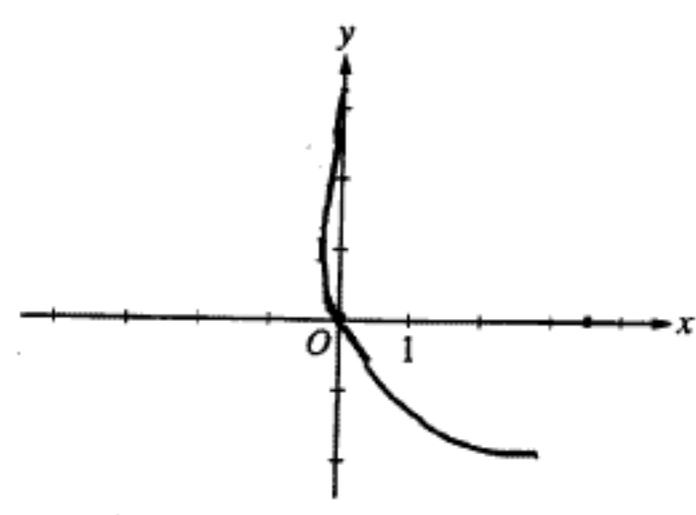
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$$x(t) = \frac{t^2}{2} - \ln(1+t) \text{ and } y(t) = 3 \sin t.$$

- (a) Sketch the path of the particle in the  $xy$ -plane below. Indicate the direction of motion along the path.

$$\frac{e^x}{e^{\frac{t^2}{2}}} = 1+t$$



F

- (b) At what time  $t$ ,  $0 \leq t \leq \pi$ , does  $x(t)$  attain its minimum value? What is the position  $(x(t), y(t))$  of the particle at this time?

$$\frac{dx}{dt} = t - \frac{1}{1+t} = 0$$

$$t = 0.618$$

$$(-0.29, 1.738)$$

- (c) At what time  $t$ ,  $0 < t < \pi$ , is the particle on the  $y$ -axis? Find the speed and the acceleration vector of the particle at this time.

$$t = 1.286$$

$$\vec{v} = (0.849, 0.843)$$

$$\vec{a} = (1.191, -2.879)$$

$$1 + \frac{1}{1+t} = -3 \sin t$$



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