



**AP[®] Calculus BC
2004 Sample Student Responses
Form B**

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CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\text{speed} = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$\frac{dx}{dt}(0) = \sqrt{0^2 + 9} = \sqrt{9} = 3 \quad \frac{dy}{dt}(0) = 2e^0 + 5e^0 = 2 + 5 = 7$$

$$\text{speed} = \sqrt{3^2 + 7^2} = \sqrt{58} = 7.616 \text{ (3d.p.)}$$

$$\text{acceleration: } \frac{d^2x}{dt^2} = \frac{1}{2\sqrt{t^2+9}} (4t^3) = \frac{2t^3}{\sqrt{t^2+9}}$$

$$\frac{d^2y}{dt^2} = 2e^t - 5e^{-t}$$

$$\left(\frac{2t^3}{\sqrt{t^2+9}}, 2e^t - 5e^{-t} \right) \Rightarrow \text{acceleration } (0, -3) \text{ at } t=0$$

Work for problem 1(b)

$$m_T = \text{slope} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2e^t + 5e^{-t}}{\sqrt{t^2+9}} \quad \text{at } t=0 \quad \boxed{\frac{dy}{dx} = \frac{7}{3}}$$

$$y - y_0 = m_T(x - x_0) \Rightarrow y - 1 = \frac{7}{3}(x - 4) \Rightarrow y - 1 = \frac{7}{3}x - \frac{28}{3}$$

$$\Rightarrow y = \frac{7}{3}x - \frac{25}{3} \Rightarrow \boxed{3y = 7x - 25}$$

Continue problem 1 on page 5.

Work for problem 1(c)

$$\begin{aligned}
 d &= \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^3 \sqrt{t^4 + 9 + (2e^t + 5e^{-t})^2} dt \\
 &= \int_0^3 \sqrt{t^4 + 9 + 4e^{2t} + 20 + 25e^{-2t}} dt \\
 &= \int_0^3 \sqrt{t^4 + 4e^{2t} + 25e^{-2t} + 29} dt \\
 &= 45.227
 \end{aligned}$$

Work for problem 1(d)

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \Rightarrow dx = \sqrt{t^4 + 9} dt \Rightarrow x = \int_0^3 \sqrt{t^4 + 9} dt.$$

Since $\frac{dx}{dt} \neq 0$ & $\frac{dy}{dt} \neq 0$, \Rightarrow particle does not stop
 \rightarrow moves in a straight line.

$$\begin{aligned}
 x\text{-coordinate at } t=3 &= x(0) + \int_0^3 \sqrt{t^4 + 9} dt \\
 &= 4 + 13.931 = 17.931.
 \end{aligned}$$

GO ON TO THE NEXT PAGE.

CALCULUS BC
SECTION II, Part A
 Time—45 minutes
 Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

acceleration vector

$$\left(\frac{1(4+3)}{2(4+9)^{\frac{1}{2}}}, 2e^t - 5e^{-t} \right)$$

$$\text{Speed} = \text{magnitude} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{(\sqrt{49})^2 + (2e^t + 5e^{-t})^2}$$

$$t=0 \rightarrow \text{speed} = \sqrt{(\sqrt{49})^2 + (2e^0 + 5e^{-0})^2}$$

$$= \sqrt{49 + 49} = \sqrt{98}$$

Work for problem 1(b)

$t=0$ point of tangency: $(4, 1)$

$$\text{slope} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2+5}{3} = \frac{7}{3}$$

$$y - 1 = \frac{7}{3}(x - 4)$$

$$y - 1 = \frac{7}{3}x - \frac{28}{3} + 1$$

$$y = \frac{7}{3}x - \frac{31}{3}$$

Continue problem 1 on page 5.

1

1

1

1

1

1

1

1

1

1

D₂

Work for problem 1(c)

Total dist ~~of~~ = length of curve

$$L = \int_0^3 \sqrt{(5t^4 + 9)^2 + (2e^t + 5e^{-t})^2}$$

$$= 45.227$$

Work for problem 1(d)

$$x(t) = \int_0^t (t^4 + 9)^{\frac{1}{2}} + C$$

~~$$x(t) = \int_0^t (t^4 + 9)^{\frac{1}{2}} + C$$~~

t=0

$$x(0) = 0 + C = 4, \quad |C=4|$$

$$x(3) = \int_0^3 (t^4 + 9)^{\frac{1}{2}} + 4 =$$

$$= 13.9307 + 4 = 17.931 \quad (\text{x coordinate})$$

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