



AP[®] Calculus BC 2001 Sample Student Responses

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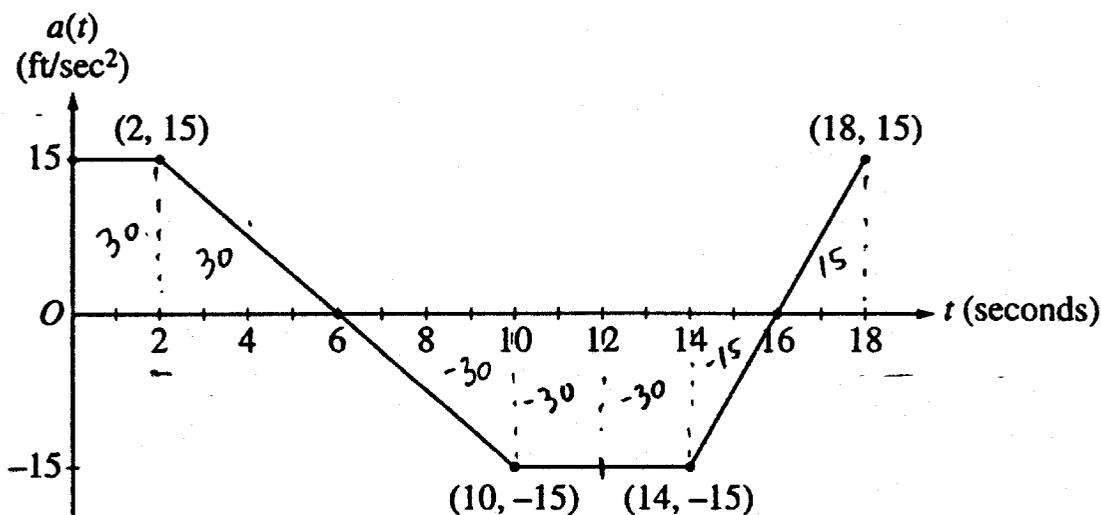
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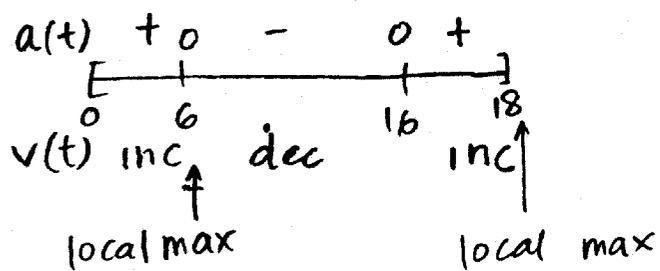
Work for problem 3(a)

Yes, the velocity is increasing at $t=2s$ because according to the graph, the car is accelerating at that time. Acceleration is positive, thus the car must be getting faster.
 $(a_{t=2} = 15 \text{ ft/s}^2)$

Work for problem 3(b)

$v = \int_0^t a \, dt$
 $\therefore \text{velocity at time } x = \text{area under graph between } t=0 \text{ and } t=x + 55$
 (initial velocity \downarrow)
 $\therefore v = 55$ when area under graph is zero
 $v = 55 \text{ ft/s}$ at $t = 12$

Work for problem 3(c)



$$\begin{aligned}
 V_6 &= \int_0^6 a(t) dt + 55 \\
 &= 60 + 55 \\
 &= 115 \text{ ft/s}
 \end{aligned}$$

$$\begin{aligned}
 V_{18} &= \int_0^{18} a(t) dt + 55 \\
 &= -30 + 55 \\
 &= 25 \text{ ft/s}
 \end{aligned}$$

∴ absolute max velocity = 115 ft/s
at it occurs at $t=6s$

Work for problem 3(d)

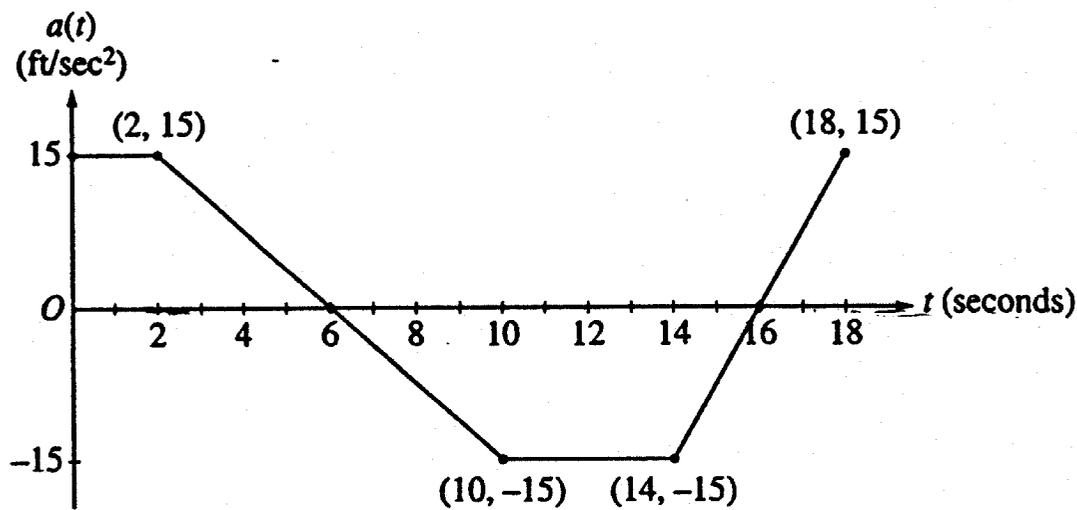
$$v = \int_0^x a(t) dt + 55$$

$$-55 = \int_0^x a(t) dt$$

for velocity to be zero, the area under the graph must be -55 , which does not occur between $t=0s$ and $t=18s$, as shown by the graph.

The lowest velocity on the interval $0 \leq t \leq 18$ is at $t=16$, and it is 10 ft/s .

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Work for problem 3(a)

At $t=2$ seconds, the acceleration of the car is positive $15 \frac{\text{ft}}{\text{sec}^2} \therefore$

yes, the velocity of the car is increasing

Work for problem 3(b)

$$v(t) = \int_0^t a(t) dt + 55 \frac{\text{ft}}{\text{sec}}$$

$$55 \frac{\text{ft}}{\text{sec}} = \int_0^x a(t) dt + 55 \frac{\text{ft}}{\text{sec}}$$

$$0 = \int_0^x a(t) dt$$

from graph $\int_0^{12} a(t) dt = 0 \therefore v(12) = 55 \frac{\text{ft}}{\text{sec}}$

$$2(15) + \frac{1}{2}(4)(15) - (2(15) + \frac{1}{2}(4)(15)) = 0$$

$$\boxed{t=12}$$

Work for problem 3(c)

$$v'(t) = a(t)$$

from graph

$$v'(t) \text{ pos. } [0, 6)$$

$$v'(t) = 0 \quad t = 6$$

$$v'(t) \text{ neg. } (6, 16) \quad \text{---}$$

$$v'(t) = 0 \quad t = 16$$

$$v'(t) \text{ pos. } (16, 18]$$

A maximum occurs at $t = 6$ because the acceleration switches from positive to negative at that point and continues to be negative for an interval longer than it is positive.

$$v(t) = \int_0^x a(t) dt + \frac{55 \text{ ft}}{\text{sec}}$$

$$v(6) = \int_0^6 a(t) dt + \frac{55 \text{ ft}}{\text{sec}}$$

$$= \frac{55 \text{ ft}}{\text{sec}} + 2 \text{ sec} \left(\frac{15 \text{ ft}}{\text{sec}^2} \right) + \frac{1}{2} (4 \text{ sec}) \left(\frac{15 \text{ ft}}{\text{sec}^2} \right) = \boxed{\frac{115 \text{ ft}}{\text{sec}}}$$

Work for problem 3(d)

$$v(t) = \int_0^x a(t) dt + \frac{55 \text{ ft}}{\text{sec}}$$

$$0 = \int_0^x a(t) dt + \frac{55 \text{ ft}}{\text{sec}}$$

$$-\frac{55 \text{ ft}}{\text{sec}} \neq \int_0^x a(t) dt \quad \text{from graph, the } \int a(t) dt \text{ never equals } \frac{55 \text{ ft}}{\text{sec}}$$

the velocity is never equal to zero