



AP Calculus BC 2000 Student Samples

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT[®], the PSAT/NMSQT[™], the Advanced Placement Program[®] (AP[®]), and Pacesetter[®]. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

Copyright © 2001 by College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, and the acorn logo are registered trademarks of the College Entrance Examination Board.

Work for problem 3(a)

$$f^0(5) = \frac{(-1)^0 0!}{2^0 (0+2)} = \frac{1}{2}$$

$$f^1(5) = \frac{(-1) \cdot 1}{2(3)} = -\frac{1}{6}$$

$$f^2(5) = \frac{\cancel{2}^2 2!}{4(4)} = \frac{1}{8}$$

$$f^3(5) = \frac{(-1) 3!}{8(5)} = \frac{-6}{40} = -\frac{3}{20}$$

$$P_3 = \frac{1}{2} - \frac{1}{6} \frac{(x-5)}{1!} + \frac{1}{8} \frac{(x-5)^2}{2!} - \frac{3}{20} \frac{(x-5)^3}{3!}$$

$$P_3 = \frac{1}{2} - \frac{1}{6}(x-5) + \frac{1}{16}(x-5)^2 - \frac{1}{40}(x-5)^3$$

Work for problem 3(b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n n!}{2^n (n+2)} \cdot \frac{(x-5)^n}{n!}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x-5)^n}{2^n (n+2)}$$

Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{(x-5)^n (x-5)}{2^n \cdot 2(n+3)} \cdot \frac{2^n (n+2)}{(x-5)^n} \right| < 1$$

$$\lim_{n \rightarrow \infty} \left| \frac{n+2}{2n+3} \cdot (x-5) \right| < 1$$

$$\frac{1}{2} |x-5| < 1$$

$$|x-5| < 2$$

Continue problem 3 on page 9.

radius of convergence = 2

Work for problem 3(c)

Alternating series, so \leftarrow remainder left off is less than first term \rightarrow $R_n \leq |a_{n+1}|$ (R=error)

$$R_6 \leq \left| \frac{(6-5)^7}{2^7(7+2)} \right|$$

$$R_6 \leq 8.68055556 \times 10^{-4}$$

$$R_6 \leq .00086805556$$

and

$$.0008680556 \leq .001$$

so the sixth degree Taylor polynomial approximates f with an error less than $\frac{1}{1000}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

$$\frac{t^n(s) \cdot (x-5)^n}{n!} = \frac{(-1)^n \cdot (x-5)^n}{2^n(n+2)}$$

$$\frac{1}{2} - \frac{(x-5)}{6} + \frac{(x-5)^2}{16} - \frac{(x-5)^3}{40}$$

Work for problem 3(b)

ratio test

$$\frac{(-1)^{n+1} \cdot (x-5)^{n+1}}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{(-1)^n(x-5)^n} = \lim_{n \rightarrow \infty} \left| \frac{(x-5)(n+2)}{2(n+3)} \right| = \frac{1}{2}$$

$$\left| \frac{x-5}{2} \right| < 1$$

$$-2 < |x-5| < 2$$

$$\boxed{3 < x < 7}$$

Continue problem 3 on page 9.

Work for problem 3(c)

6th degree term

$$\frac{(x-5)^6}{512} \Rightarrow \frac{1}{512}$$

$x=6$

next term

$$\frac{(x-5)^7}{1152} = \frac{1}{1152}$$

$x=6$

a sixth degree Taylor polynomial will approximate $f(6)$ with an error less than $\frac{1}{1000}$ because every term after it has a value of less than $\frac{1}{1000}$ for $f(6)$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a) centered at $x=5$ so $c=5$

$$P(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \dots$$

$$P(x) = 5 + \frac{(-1)^1(1!)}{2^1(1+2)}(x-5) + \frac{(-1)^2(2!)}{2^2(2+2)} \cdot \frac{(x-5)^2}{2!} + \frac{(-1)^3(3!)}{2^3(3+2)} \frac{(x-5)^3}{3!} + \dots$$

$$P(x) = 5 - \frac{(x-5)}{6} + \frac{(x-5)^2}{16} - \frac{(x-5)^3}{40} + \dots$$

n^{th} term given by $\frac{(-1)^n n!}{2^n(n+2)} \cdot \frac{(x-5)^n}{n!} = \frac{(-1)^n (x-5)^n}{2^n(n+2)}$

Work for problem 3(b)

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} (x-5)^{n+1}}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{(-1)^n (x-5)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x-5)(n+2)}{2(n+3)} \right| = \left| \frac{x-5}{2} \right| \lim_{n \rightarrow \infty} \frac{n+2}{n+3} =$$

$\left| \frac{x-5}{2} \right| < 1$ by ratio test, if $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$, series converges

$$|x-5| < 2$$

$$x-5 = 2 \quad \text{or} \quad 5-x = 2$$

$$x = 7 \quad \text{or} \quad x = 3$$

$$3 < x < 7$$

∴ the radius of convergence for this series is 7-3 or 4.

Continue problem 3 on page 9.

Work for problem 3(c)

$$n^{\text{th}} \text{ term is } \frac{(-1)^n (x-5)^n}{2^n (n+2)}$$

because $P(x)$ is an alternating series, the alternating series remainder theorem guarantees that error for n terms will be less than or equal to the $(n+1)$ term

for a 6th degree polynomial, the error is $\leq |7^{\text{th}} \text{ term}|$

$$\text{error} \leq \left| \frac{(-1)^7 (x-5)^7}{2^7 (n+2)} \right| \quad x=6$$

$$\text{error} \leq \left| \frac{(-1)^7 (6-5)^7}{2^7 (6+2)} \right|$$

$$\text{error} \leq \frac{1}{1024}$$

$$\therefore \frac{1}{1024} < \frac{1}{1000}$$

\therefore the error for a 6th degree polynomial is less than $\frac{1}{1000}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.