



AP[®] Calculus BC 2002 Free-Response Questions

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2002 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. Let f and g be the functions given by $f(x) = e^{-x}$ and $g(x) = \ln x$.
- (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.
- (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.
- (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$. Show the analysis that leads to your answers.
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2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

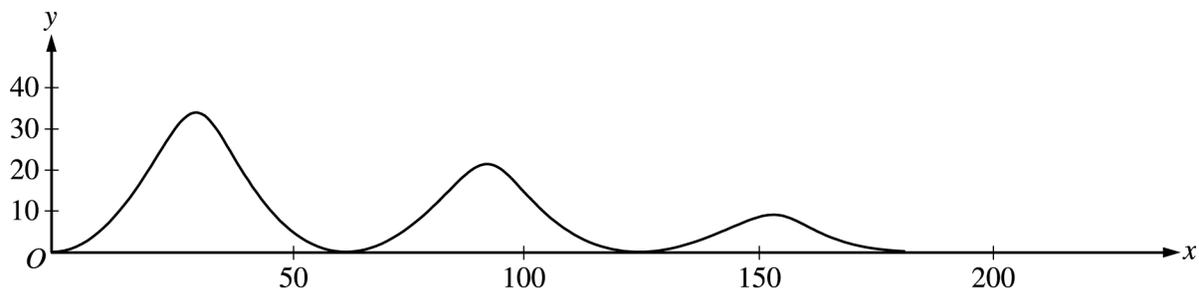
The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \leq t \leq 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

- (a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.
- (b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
- (c) Let $H(t) = \int_9^t (E(x) - L(x)) dx$ for $9 \leq t \leq 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.
- (d) At what time t , for $9 \leq t \leq 23$, does the model predict that the number of people in the park is a maximum?
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3. The figure above shows the path traveled by a roller coaster car over the time interval $0 \leq t \leq 18$ seconds. The position of the car at time t seconds can be modeled parametrically by

$$x(t) = 10t + 4 \sin t$$

$$y(t) = (20 - t)(1 - \cos t),$$

where x and y are measured in meters. The derivatives of these functions are given by

$$x'(t) = 10 + 4 \cos t$$

$$y'(t) = (20 - t) \sin t + \cos t - 1.$$

- Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.
- Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.
- Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.
- For $0 < t < 18$, there are two times at which the car is at ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

END OF PART A OF SECTION II

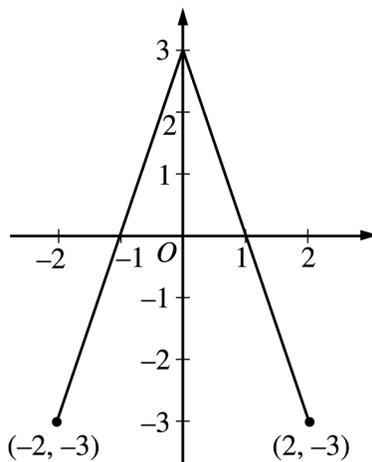
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CALCULUS BC
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

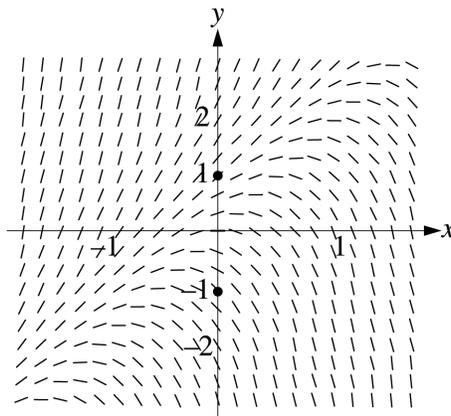
4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.
 - (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.
 - (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.
 - (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
(Note: The axes are provided in the pink test booklet only.)

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5. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

- (a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the slope field provided in the pink test booklet.)



- (b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.
- (c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.
- (d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

6. The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

- (a) Find the interval of convergence of the Maclaurin series for f . Justify your answer.
- (b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.
- (c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

END OF EXAMINATION