



## **AP<sup>®</sup> Calculus BC 2011 Free-Response Questions Form B**

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2011 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC  
SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function  $S$ , where  $S(t)$  is measured in millimeters and  $t$  is measured in days for  $0 \leq t \leq 60$ . The rate at which the height of the water is rising in the can is given by  $S'(t) = 2\sin(0.03t) + 1.5$ .
- (a) According to the model, what is the height of the water in the can at the end of the 60-day period?
- (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.
- (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time  $t = 7$ ? Indicate units of measure.
- (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function  $M$ , where  $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$ . The height  $M(t)$  is measured in millimeters, and  $t$  is measured in days for  $0 \leq t \leq 60$ . Let  $D(t) = M'(t) - S'(t)$ . Apply the Intermediate Value Theorem to the function  $D$  on the interval  $0 \leq t \leq 60$  to justify that there exists a time  $t$ ,  $0 < t < 60$ , at which the heights of water in the two cans are changing at the same rate.
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2. The polar curve  $r$  is given by  $r(\theta) = 3\theta + \sin \theta$ , where  $0 \leq \theta \leq 2\pi$ .
- (a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of  $r$ .
- (b) For  $\frac{\pi}{2} \leq \theta \leq \pi$ , there is one point  $P$  on the polar curve  $r$  with  $x$ -coordinate  $-3$ . Find the angle  $\theta$  that corresponds to point  $P$ . Find the  $y$ -coordinate of point  $P$ . Show the work that leads to your answers.
- (c) A particle is traveling along the polar curve  $r$  so that its position at time  $t$  is  $(x(t), y(t))$  and such that  $\frac{d\theta}{dt} = 2$ . Find  $\frac{dy}{dt}$  at the instant that  $\theta = \frac{2\pi}{3}$ , and interpret the meaning of your answer in the context of the problem.
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WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

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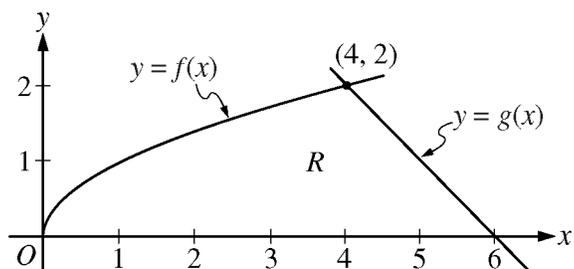
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**CALCULUS BC**  
**SECTION II, Part B**

**Time—60 minutes**

**Number of problems—4**

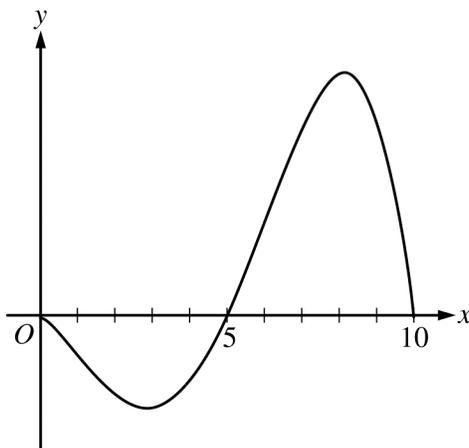
**No calculator is allowed for these problems.**



3. The functions  $f$  and  $g$  are given by  $f(x) = \sqrt{x}$  and  $g(x) = 6 - x$ . Let  $R$  be the region bounded by the  $x$ -axis and the graphs of  $f$  and  $g$ , as shown in the figure above.
- Find the area of  $R$ .
  - The region  $R$  is the base of a solid. For each  $y$ , where  $0 \leq y \leq 2$ , the cross section of the solid taken perpendicular to the  $y$ -axis is a rectangle whose base lies in  $R$  and whose height is  $2y$ . Write, but do not evaluate, an integral expression that gives the volume of the solid.
  - There is a point  $P$  on the graph of  $f$  at which the line tangent to the graph of  $f$  is perpendicular to the graph of  $g$ . Find the coordinates of point  $P$ .

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Graph of  $f$

4. The graph of the differentiable function  $y = f(x)$  with domain  $0 \leq x \leq 10$  is shown in the figure above. The area of the region enclosed between the graph of  $f$  and the  $x$ -axis for  $0 \leq x \leq 5$  is 10, and the area of the region enclosed between the graph of  $f$  and the  $x$ -axis for  $5 \leq x \leq 10$  is 27. The arc length for the portion of the graph of  $f$  between  $x = 0$  and  $x = 5$  is 11, and the arc length for the portion of the graph of  $f$  between  $x = 5$  and  $x = 10$  is 18. The function  $f$  has exactly two critical points that are located at  $x = 3$  and  $x = 8$ .
- (a) Find the average value of  $f$  on the interval  $0 \leq x \leq 5$ .
- (b) Evaluate  $\int_0^{10} (3f(x) + 2) dx$ . Show the computations that lead to your answer.
- (c) Let  $g(x) = \int_5^x f(t) dt$ . On what intervals, if any, is the graph of  $g$  both concave up and decreasing? Explain your reasoning.
- (d) The function  $h$  is defined by  $h(x) = 2f\left(\frac{x}{2}\right)$ . The derivative of  $h$  is  $h'(x) = f'\left(\frac{x}{2}\right)$ . Find the arc length of the graph of  $y = h(x)$  from  $x = 0$  to  $x = 20$ .

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$t$ (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function  $B$  models Ben's position on the track, measured in meters from the western end of the track, at time  $t$ , measured in seconds from the start of the ride. The table above gives values for  $B(t)$  and Ben's velocity,  $v(t)$ , measured in meters per second, at selected times  $t$ .
- (a) Use the data in the table to approximate Ben's acceleration at time  $t = 5$  seconds. Indicate units of measure.
- (b) Using correct units, interpret the meaning of  $\int_0^{60} |v(t)| dt$  in the context of this problem. Approximate  $\int_0^{60} |v(t)| dt$  using a left Riemann sum with the subintervals indicated by the data in the table.
- (c) For  $40 \leq t \leq 60$ , must there be a time  $t$  when Ben's velocity is 2 meters per second? Justify your answer.
- (d) A light is directly above the western end of the track. Ben rides so that at time  $t$ , the distance  $L(t)$  between Ben and the light satisfies  $(L(t))^2 = 12^2 + (B(t))^2$ . At what rate is the distance between Ben and the light changing at time  $t = 40$ ?

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6. Let  $f(x) = \ln(1 + x^3)$ .

- (a) The Maclaurin series for  $\ln(1 + x)$  is  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \cdot \frac{x^n}{n} + \dots$ . Use the series to write the first four nonzero terms and the general term of the Maclaurin series for  $f$ .
- (b) The radius of convergence of the Maclaurin series for  $f$  is 1. Determine the interval of convergence. Show the work that leads to your answer.
- (c) Write the first four nonzero terms of the Maclaurin series for  $f'(t^2)$ . If  $g(x) = \int_0^x f'(t^2) dt$ , use the first two nonzero terms of the Maclaurin series for  $g$  to approximate  $g(1)$ .
- (d) The Maclaurin series for  $g$ , evaluated at  $x = 1$ , is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from  $g(1)$  by less than  $\frac{1}{5}$ .

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**END OF EXAM**