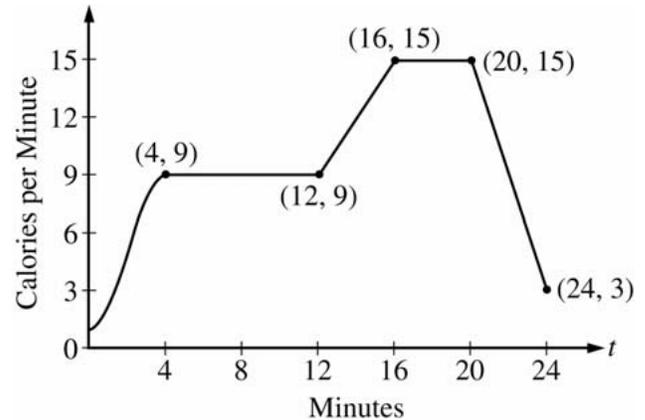


AP[®] CALCULUS BC
2006 SCORING GUIDELINES (Form B)

Question 4

The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f . In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for

$0 \leq t \leq 4$ and f is piecewise linear for $4 \leq t \leq 24$.



- (a) Find $f'(22)$. Indicate units of measure.
- (b) For the time interval $0 \leq t \leq 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.
- (c) Find the total number of calories burned over the time interval $6 \leq t \leq 18$ minutes.
- (d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \leq t \leq 18$.

(a) $f'(22) = \frac{15 - 3}{20 - 24} = -3$ calories/min/min

(b) f is increasing on $[0, 4]$ and on $[12, 16]$.

On $(12, 16)$, $f'(t) = \frac{15 - 9}{16 - 12} = \frac{3}{2}$ since f has constant slope on this interval.

On $(0, 4)$, $f'(t) = -\frac{3}{4}t^2 + 3t$ and

$f''(t) = -\frac{3}{2}t + 3 = 0$ when $t = 2$. This is where f' has a maximum on $[0, 4]$ since $f'' > 0$ on $(0, 2)$ and $f'' < 0$ on $(2, 4)$.

On $[0, 24]$, f is increasing at its greatest rate when $t = 2$ because $f'(2) = 3 > \frac{3}{2}$.

(c) $\int_6^{18} f(t) dt = 6(9) + \frac{1}{2}(4)(9 + 15) + 2(15)$
 $= 132$ calories

(d) We want $\frac{1}{12} \int_6^{18} (f(t) + c) dt = 15$.

This means $132 + 12c = 15(12)$. So, $c = 4$.

OR

Currently, the average is $\frac{132}{12} = 11$ calories/min.

Adding c to $f(t)$ will shift the average by c .

So $c = 4$ to get an average of 15 calories/min.

1 : $f'(22)$ and units

4 : $\begin{cases} 1 : f' \text{ on } (0, 4) \\ 1 : \text{shows } f' \text{ has a max at } t = 2 \text{ on } (0, 4) \\ 1 : \text{shows for } 12 < t < 16, f'(t) < f'(2) \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{method} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{setup} \\ 1 : \text{value of } c \end{cases}$

4

4

4

4

4

4

4

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4

4A

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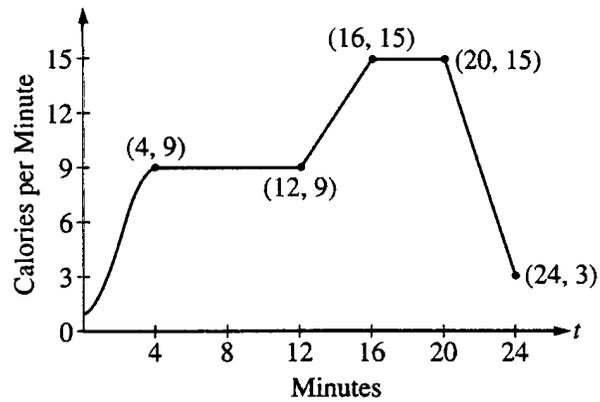
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$\begin{aligned}
 f'(22) &= \text{Gradient of straight line from } t=20 \text{ to } t=24 \\
 &= \frac{3-15}{24-20} \\
 &= \frac{-12}{4} \\
 &= -3 \text{ calories/minute}^2
 \end{aligned}$$

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Continue problem 4 on page 11.

Work for problem 4(b)

From graph, we see that f is only increasing in the interval $0 \leq t \leq 4$ and $12 \leq t \leq 16$

Rate of increment for $12 \leq t \leq 16$

$$= \frac{15-9}{16-12}$$

$$= \frac{6}{4} = 1.5$$

For $0 \leq t \leq 4$,

$$f(t) = -\frac{3}{4}t^2 + 3t$$

$$f'(t) = -\frac{3}{2}t + 3$$

At greatest rate of increment, $f''(t) = 0 \Rightarrow t = 2$

$f''(t) = -\frac{3}{2} < 0 \Rightarrow$ At $t = 2$, rate of increment is greatest and not smallest

$$f'(2) = -\frac{3}{2}(2)^2 + 3(2) = 3 > 1.5$$

$\therefore f$ is increasing at its greatest rate ~~from~~ at $t = 2$

Work for problem 4(c)

From graph
 For $6 \leq t \leq 12$,

$$\text{Total number of calories burned} = \int_6^{18} f(t) dt$$

$$= 9(12-6) + \frac{1}{2}(9+15)(16-12) + 15(18-16)$$

$$= 54 + 48 + 30 = 132 \text{ calories}$$

Work for problem 4(d)

Before setting is changed, average calories in $6 \leq t \leq 18$

$$= \frac{1}{18-6} \int_6^{18} f(t) dt = \frac{132}{12} = 11 \text{ calories}$$

$$\text{Now, } \frac{1}{18-6} \int_6^{18} [f(t) + c] dt = 15$$

$$\Rightarrow \frac{1}{12} \int_6^{18} f(t) dt + \frac{1}{12} \int_6^{18} c dt = 15$$

$$\Rightarrow \frac{1}{12} [c \times 12] = 15 - 11 = 4$$

$$\frac{1}{2}(18-6)c = 4$$

$$\Rightarrow c = 4$$

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4

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4B

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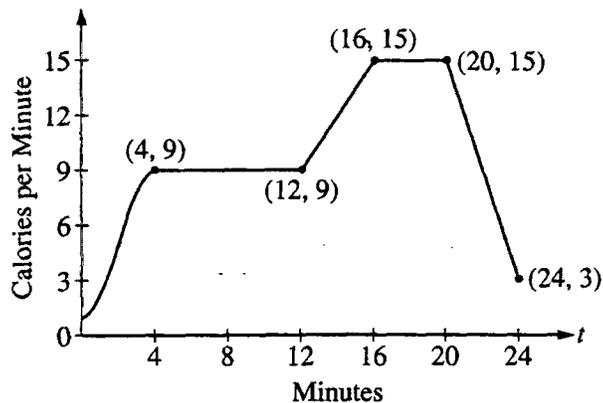
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$f'(22) = \frac{f(24) - f(20)}{24 - 20} = \frac{3 - 15}{4} = -\frac{12}{4} = -3$$

$$\therefore -3 \text{ cal/min}^2$$

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Continue problem 4 on page 11

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$\begin{aligned}
 \text{i) } 0 \leq t \leq 4, \quad f'(t) &= -\frac{3}{4}t^2 + 3t \\
 &= -\frac{3}{4}(t^2 - 4t + 4) + 3 \\
 &= -\frac{3}{4}(t-2)^2 + 3
 \end{aligned}$$

$$\text{ii) } f'(t) = 0 \quad \text{for } 4 \leq t < 12$$

$$\text{iii) } f'(t) = \frac{15-9}{16-12} = \frac{6}{4} = \frac{3}{2} \quad \text{for } 12 \leq t < 16$$

$$\text{iv) } f'(t) = 0 \quad \text{for } 16 \leq t < 20$$

$$\text{v) } f'(t) < 0 \quad \text{for } 20 \leq t < 24$$

$\therefore f'(t)$ is the greatest when
 $t=2$,

which also means that

$f(t)$ is increasing at
its greatest rate

$$\therefore t=2$$

Work for problem 4(c)

$$\text{i) } 6 \leq t < 12 \quad f(t) = 9 \quad 6 \times 9 = 54$$

$$\text{ii) } 12 \leq t < 16 \quad \frac{1}{2} \times (9+15) \times 4 = 48$$

$$\text{iii) } 16 \leq t \leq 18 \quad f(t) = 15 \quad 2 \times 15 = 30$$

$$\therefore 54 + 48 + 30 = 132 \quad \therefore 132 \text{ calories}$$

Work for problem 4(d)

$$132 + c \times (18-6) = 180 + 12c$$

$$\text{Since } \frac{132 + 12c}{12} = 15, \quad c = \frac{180 - 132}{12} = 4$$

$$\therefore c = 4$$

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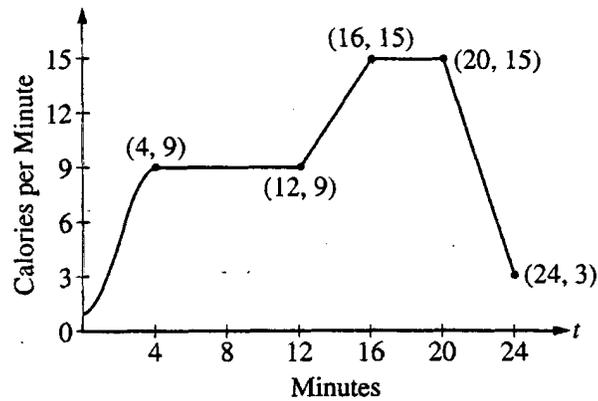
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CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

function for $20 \leq x \leq 24$

$$m = \frac{3-15}{24-20} \quad y-3 = -3(x-24)$$

$$\frac{-12}{4} \quad y = -3x + 75$$

$$m = -3$$

$$f(x) = -3x + 75$$

$$f'(22) = -3 \text{ calories/minutes}^2$$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

function increases at $0 \leq x < 4$, $12 \leq x \leq 16$

$$f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$$

$$f'(t) = -\frac{3}{4}t^2 + 3t$$

$$f''(t) = -\frac{3}{2}t + 3 = 0$$

$$t = 2$$

maximum for $f'(t)$

$$\text{at } t = 2$$

$$f'(2) = -\frac{3}{4}(4) + 6$$

$$= 3$$

$$m = \frac{15-9}{16-12} = \frac{3}{2}$$

$$f(x) = y - 15 = \frac{3}{2}(x-16)$$

$$f(x) = \frac{3}{2}x - 9$$

$$f'(x) = \frac{3}{2}$$

Work for problem 4(c)

$$\text{Calories total} = \int_6^{12} 9 dx + \int_{12}^{16} \left(\frac{3}{2}x - 9\right) dx + \int_{16}^{18} 15 dx$$

$$9x \Big|_6^{12} + \left[\frac{3}{4}x^2 - 9x\right]_{12}^{16} + 15x \Big|_{16}^{18}$$

$$108 - 54 + 183 - 99 + 270 - 240$$

$$54 + 30 + 84$$

164 calories burned

Work for problem 4(d)

$$15 = \frac{\int_6^{18} (f(x) + c) dx}{12}$$

$$180 = \int_6^{18} f(x) dx + \int_6^{18} c dx$$

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AP[®] CALCULUS BC
2006 SCORING COMMENTARY (Form B)

Question 4

Overview

This problem presented students with a piecewise-defined function f that modeled the rate at which a person using an exercise machine burns calories. The graph of f consisted of a cubic part and a part that was piecewise linear. In part (a) students were asked to find $f'(22)$, which required them to recognize the relationship between this value and the slope of one of the line segments in the graph of f . It was also important to use correct units. For part (b) students had to consider the two parts of the graph where f was increasing and determine the time when f was increasing at its greatest rate. In part (c) students had to use a definite integral to find the total number of calories burned over a given time interval. The evaluation of the definite integral could be done using geometry since the graph over the given time interval consisted of two line segments, one of which was horizontal. For part (d) students were expected to use the value of their integral from part (c) to work with the average value of the function f shifted up by c calories per minute.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d). The student shows correct work for parts (a), (c), and (d). In part (b) the first derivative is correct and the student earned the first point. The student neither explains why $f'(t)$ has a maximum at 2 nor states the value of $f'(2)$. A proper reasoning for the final answer is not given, and the student did not earn any more points.

Sample: 4C
Score: 4

The student earned 4 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). The work in part (a) is correct. In part (b) the first derivative is correct, so the student earned the first point. The student neither explains why $f'(t)$ has a maximum at 2 in the interval $0 \leq t \leq 4$ nor shows a comparison among the values of $f'(t)$. The student does not provide a final answer and did not earn any more points. In part (c) the setup is correct. The antiderivative of the second integral is incorrect, so the student did not earn the answer point. In part (d) the setup is correct, but the student does not finish the problem and could not earn the second point.