



AP[®] Calculus BC 2002 Sample Student Responses Form B

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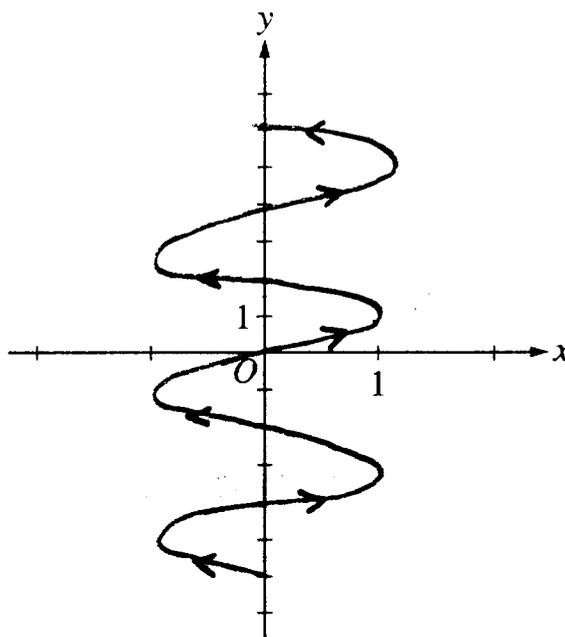
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CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)



Work for problem 1(b)

Range of $x(t)$: $-1 \leq x \leq 1$
 Range of $y(t)$: $-2\pi \leq y \leq 2\pi$

Work for problem 1(c)

$$x(t) = \sin(3t)$$

$$\frac{dx}{dt} = 3\cos(3t)$$

$$\text{Cr. point: } 3\cos(3t) = 0$$

$$\cos(3t) = 0$$

$$3t = \pi/2$$

$$\boxed{t = \pi/6}$$

$$\frac{d^2x}{dt^2} = -9\sin(3t)$$

$$\frac{d^2x}{dt^2} \Big|_{t=\pi/6} = -9\sin(\pi/2) < 0 \Rightarrow \boxed{t = \pi/6 \text{ is at a local maximum}}$$

$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\frac{dx}{dt} \Big|_{t=\pi/6} = 3\cos(\pi/2) = 0$$

$$\frac{dy}{dt} = 2 \text{ for all } t$$

$$\text{Speed at } \pi/6 = \sqrt{0 + 2^2} = \underline{\underline{2}} \text{ units/}$$

Work for problem 1(d)

$$\text{Distance travelled} = \int_{-\pi}^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$= \int_{-\pi}^{\pi} \sqrt{9\cos^2(3t) + 4} dt \approx \underline{\underline{17.973}}$$

$$5\pi \approx 15.708$$

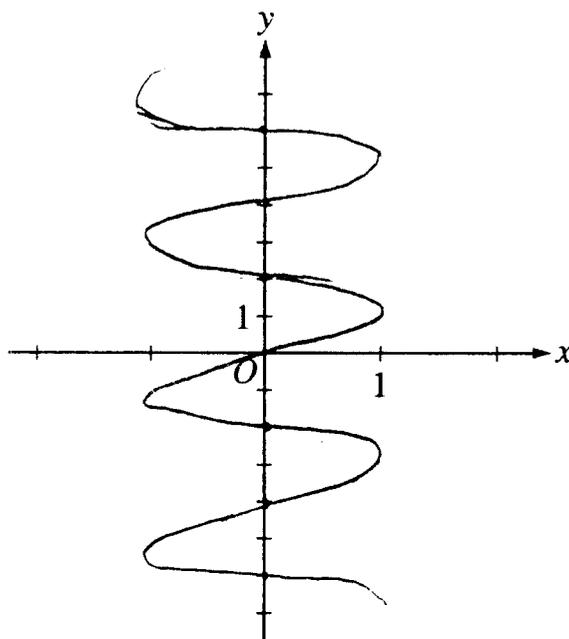
\therefore Distance traveled from $t = -\pi$ to $t = \pi$ is greater than 5π .

GO ON TO THE NEXT PAGE.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)



Work for problem 1(b)

$$x(t) = \sin(3t) \quad \text{with note } -1 \leq \sin < 1$$

$$\boxed{-1 \leq x \leq 1}$$

$$y(t) = 2t$$

$$\boxed{y - \text{all real numbers}}$$

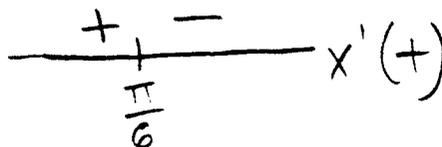
Work for problem 1(c)

$$x'(t) = 3 \cos(3t)$$

$$3 \cos(3t) = 0$$

$$3t = \frac{\pi}{2} \text{ or multiple}$$

$$t = \frac{\pi}{6} \text{ - smallest}$$



$$\text{speed} = \sqrt{(y')^2 + (x')^2} = \sqrt{2^2 + 9 \cos^2(3t)}$$

$$= \sqrt{4 + 0}$$

$$= 2 \text{ un/time unit}$$

Work for problem 1(d)

$$\text{dist} = \int_{-\pi}^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$= \int_{-\pi}^{\pi} \sqrt{9 \cos^2(3t) + 4} dt$$

$$= 17.973 \text{ un}$$

$$5\pi = 15.708 < 17.973$$

yes

GO ON TO THE NEXT PAGE.