

AP[®] CALCULUS BC
2008 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the xy -plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \quad \text{and} \quad \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

- (a) Find the acceleration vector at time $t = 4$.
 (b) Find the y -coordinate of the position of the particle at time $t = 0$.
 (c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?
 (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.

(a) $a(4) = \langle x''(4), y''(4) \rangle = \langle 0.433, -11.872 \rangle$

1 : answer

(b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601

3 : $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{uses } y(4) = 5 \\ 1 : \text{answer} \end{array} \right.$

(c) Speed = $\sqrt{(x'(t))^2 + (y'(t))^2}$
 $= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$

3 : $\left\{ \begin{array}{l} 1 : \text{expression for speed} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{array} \right.$

The particle first reaches this speed when
 $t = 2.225$ or 2.226 .

(d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} dt = 13.182$

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

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Form B
BC I
1A₁* CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\text{acceleration} = \langle x''(t), y''(t) \rangle$$

$$x''(t) = \frac{3}{2} (3t)^{-1/2} = \frac{3}{2} ((3)(4))^{-1/2} = 0.433$$

$$y''(t) = -11.872 \quad \boxed{\text{acceleration vector} = \langle 0.433, -11.872 \rangle}$$

Work for problem 1(b)

$$y(0) = y(4) + \int_4^0 3 \cos\left(\frac{t^2}{2}\right) dt$$

$$y(0) = 5 + \int_4^0 3 \cos\left(\frac{t^2}{2}\right) dt = \boxed{1.601}$$

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Continue problem 1 on page 5.

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1A2

Work for problem 1(c)

$$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$$

$$t = 2.226$$

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Work for problem 1(d)

$$\int_0^u \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} dt = 13.182$$

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Form B
BC I
131

* CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\frac{dx}{dt} = \sqrt{3t} \quad \frac{dy}{dt} = 3 \cos\left(\frac{t^2}{2}\right)$$

acceleration: $(x''(t), y''(t))$

$$x''(t) = \sqrt{3} \times \frac{1}{2} \times \frac{1}{\sqrt{t}} \quad y''(t) = -3 \sin\left(\frac{t^2}{2}\right) \times \frac{1}{2} \times 2t$$

$$= \frac{\sqrt{3}}{2\sqrt{t}} \quad = -3t \sin\left(\frac{t^2}{2}\right)$$

t=4

$$x''(4) = \frac{\sqrt{3}}{2\sqrt{4}}$$

$$= \frac{\sqrt{3}}{4}$$

$$\approx 0.433$$

$$y''(4) = -3 \sin\left(\frac{4^2}{2}\right)$$

$$= -3 \sin 8$$

$$\approx -2.968$$

(0.433, -2.968)

Work for problem 1(b)

$$\frac{dx}{dt} = \sqrt{3t}$$

$$\int \frac{dx}{dt} dt = \int \sqrt{3t} dt$$

$$x = \sqrt{3} t^{\frac{3}{2}} \times \frac{2}{3} + C$$

$$\frac{dy}{dt} = 3 \cos\left(\frac{t^2}{2}\right)$$

$$\int \frac{dy}{dt} dt = \int 3 \cos\left(\frac{t^2}{2}\right) dt$$

$$y = \int 3 \cos\left(\frac{t^2}{2}\right) dt$$

$$\int_0^4 3 \cos\left(\frac{t^2}{2}\right) dt = y(4) - y(0)$$

$$= 5 - y(0)$$

$$y(0) = 5 - \int_0^4 3 \cos\left(\frac{t^2}{2}\right) dt$$

$$\approx 5 - 3.399$$

$$= 1.601$$

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1B₂

Work for problem 1(c)

$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{(\sqrt{3t})^2 + \left(3\cos\left(\frac{t^2}{5}\right)\right)^2}$$

$$= \sqrt{3t + 9\cos^2\left(\frac{t^2}{5}\right)}$$

$$= 3.5$$

$$0 \leq t \leq 4$$

$$t = 2.226$$

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Work for problem 1(d)

$$\sqrt{\left(\int_0^4 \sqrt{3t} dt\right)^2 + \left(\int_0^4 3\cos\left(\frac{t^2}{5}\right) dt\right)^2}$$

$$\approx \sqrt{(9.238)^2 + (3.399)^2}$$

$$\approx 9.843$$

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Form B
BC1
1C1CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$a_{\text{acceleration}} = (0.14, -11.87)$$

$$(v)' = a$$

$$\left(\frac{dx}{dt}\right)' = \left(\sqrt{3-t}\right)'$$

$$\left(\sqrt{3-t}\right)' = \frac{1}{2}(3-t)^{-\frac{1}{2}}(-1) = -\frac{1}{2\sqrt{3-t}}$$

$$a = \frac{3}{2\sqrt{3-t}} \quad (t=4) \quad a_x = \frac{3}{2\sqrt{12}} \approx 0.14$$

$$\left(\frac{dy}{dt}\right)' = \left(3\cos\left(\frac{t^2}{2}\right)\right)' = -3t\sin\left(\frac{t^2}{2}\right)$$

$$t=4, a_y = -12\sin(8)$$

$$\text{Acceleration} = (0.14, -11.87)$$

Work for problem 1(b)

$$y_{\text{position}} = \int 3\cos\left(\frac{t^2}{2}\right) dt =$$

$$= 3 \int \cos\left(\frac{t^2}{2}\right) dt$$

$$= 3 \left\{ \frac{\sin\left(\frac{t^2}{2}\right)}{t} \right\} + C$$

$$t=0, \text{ such that } t=0$$

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Continue problem 1 on page 5.

Work for problem 1(c)

$$\text{speed} = \sqrt{(\sqrt{3}t)^2 + 3\cos\left(\frac{t^2}{2}\right)^2}$$

$$3.5 = \sqrt{(\sqrt{3}t)^2 + 3\cos\left(\frac{t^2}{2}\right)^2}$$

$$12.25 = 3t + \left(3\cos\left(\frac{t^2}{2}\right)\right)^2$$

$$\Leftrightarrow 12.25 = 3t + 9\cos^2\left(\frac{t^2}{2}\right)$$

$$\text{graph: } 3t + 9\cos^2\left(\frac{t^2}{2}\right) - 12.25$$

at 2.22 second, the speed of the particle first reach 3.5

Work for problem 1(d)

$$\text{Total distance} = \int_0^4 \sqrt{(\sqrt{3}t)^2 + \left(3\cos\left(\frac{t^2}{2}\right)\right)^2} dt$$

$$\Leftrightarrow \int_0^4 \sqrt{3t^2 + 9\cos^2\left(\frac{t^2}{2}\right)}$$

$$\approx 19.48$$

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AP[®] CALCULUS BC
2008 SCORING COMMENTARY (Form B)

Question 1

Sample: 1A

Score: 9

The student earned all 9 points.

Sample: 1B

Score: 6

The student earned 6 points: no points in part (a), 3 points in part (b), 3 points in part (c), and no points in part (d). The student presents correct work in parts (b) and (c). In part (a) the student has the correct value of $x''(4)$ but loses a factor of t in $y''(4)$, and thus the response did not earn the point. In part (d) the student does not integrate the speed in order to determine the total distance traveled.

Sample: 1C

Score: 4

The student earned 4 points: no points in part (a), 1 point in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student is not able to determine $x''(4)$. In part (b) the student has the correct integrand and earned 1 point. The student does not use the initial condition of $y(4) = 5$ so was not eligible for the answer point. In part (c) the student finds the correct equation and earned the first 2 points. The answer is only given to two decimal places, so the third point was not earned. In part (d) the student has the correct integral to determine the distance traveled and earned the first point; however, the answer is not correct, so the answer point was not earned.