



## AP<sup>®</sup> Calculus BC 2003 Free-Response Questions

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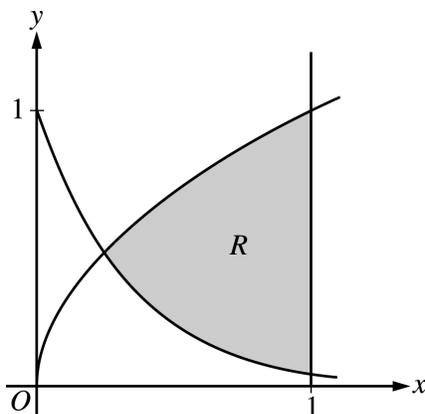
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# 2003 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

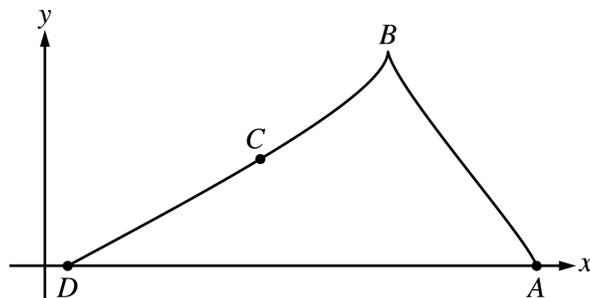
A graphing calculator is required for some problems or parts of problems.

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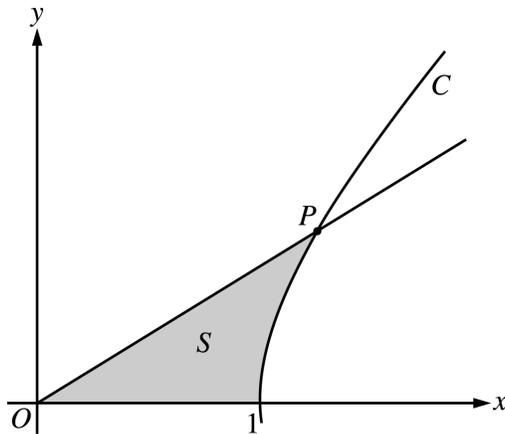
1. Let  $R$  be the shaded region bounded by the graphs of  $y = \sqrt{x}$  and  $y = e^{-3x}$  and the vertical line  $x = 1$ , as shown in the figure above.
    - (a) Find the area of  $R$ .
    - (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 1$ .
    - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a rectangle whose height is 5 times the length of its base in region  $R$ . Find the volume of this solid.
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2. A particle starts at point  $A$  on the positive  $x$ -axis at time  $t = 0$  and travels along the curve from  $A$  to  $B$  to  $C$  to  $D$ , as shown above. The coordinates of the particle's position  $(x(t), y(t))$  are differentiable functions of  $t$ , where  $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$  and  $y'(t) = \frac{dy}{dt}$  is not explicitly given. At time  $t = 9$ , the particle reaches its final position at point  $D$  on the positive  $x$ -axis.
- At point  $C$ , is  $\frac{dy}{dt}$  positive? At point  $C$ , is  $\frac{dx}{dt}$  positive? Give a reason for each answer.
  - The slope of the curve is undefined at point  $B$ . At what time  $t$  is the particle at point  $B$ ?
  - The line tangent to the curve at the point  $(x(8), y(8))$  has equation  $y = \frac{5}{9}x - 2$ . Find the velocity vector and the speed of the particle at this point.
  - How far apart are points  $A$  and  $D$ , the initial and final positions, respectively, of the particle?
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3. The figure above shows the graphs of the line  $x = \frac{5}{3}y$  and the curve  $C$  given by  $x = \sqrt{1 + y^2}$ . Let  $S$  be the shaded region bounded by the two graphs and the  $x$ -axis. The line and the curve intersect at point  $P$ .
- Find the coordinates of point  $P$  and the value of  $\frac{dx}{dy}$  for curve  $C$  at point  $P$ .
  - Set up and evaluate an integral expression with respect to  $y$  that gives the area of  $S$ .
  - Curve  $C$  is a part of the curve  $x^2 - y^2 = 1$ . Show that  $x^2 - y^2 = 1$  can be written as the polar equation  $r^2 = \frac{1}{\cos^2 \theta - \sin^2 \theta}$ .
  - Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle  $\theta$  that represents the area of  $S$ .
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END OF PART A OF SECTION II

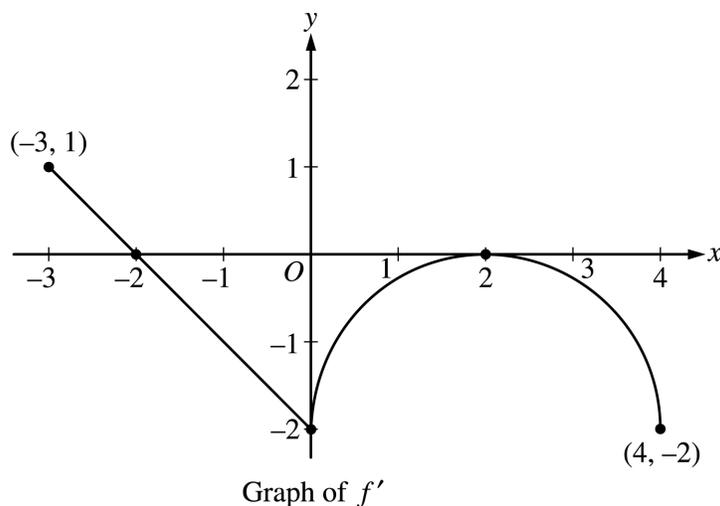
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## CALCULUS BC SECTION II, Part B

Time—45 minutes

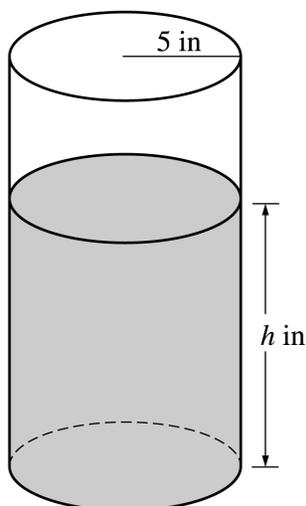
Number of problems—3

No calculator is allowed for these problems.



4. Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle, as shown above.
- On what intervals, if any, is  $f$  increasing? Justify your answer.
  - Find the  $x$ -coordinate of each point of inflection of the graph of  $f$  on the open interval  $-3 < x < 4$ . Justify your answer.
  - Find an equation for the line tangent to the graph of  $f$  at the point  $(0, 3)$ .
  - Find  $f(-3)$  and  $f(4)$ . Show the work that leads to your answers.
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5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let  $h$  be the depth of the coffee in the pot, measured in inches, where  $h$  is a function of time  $t$ , measured in seconds. The volume  $V$  of coffee in the pot is changing at the rate of  $-5\pi\sqrt{h}$  cubic inches per second. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)
- (a) Show that  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ .
- (b) Given that  $h = 17$  at time  $t = 0$ , solve the differential equation  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$  for  $h$  as a function of  $t$ .
- (c) At what time  $t$  is the coffeepot empty?

6. The function  $f$  is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers  $x$ .

- (a) Find  $f'(0)$  and  $f''(0)$ . Determine whether  $f$  has a local maximum, a local minimum, or neither at  $x = 0$ . Give a reason for your answer.
- (b) Show that  $1 - \frac{1}{3!}$  approximates  $f(1)$  with error less than  $\frac{1}{100}$ .
- (c) Show that  $y = f(x)$  is a solution to the differential equation  $xy' + y = \cos x$ .

**END OF EXAMINATION**