



AP[®] Calculus BC 2001 Sample Student Responses

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NO CALCULATOR ALLOWED

A,

Work for problem 5(a)

$$f'(x) = -3x f(x)$$

$$\lim_{b \rightarrow \infty} \int_1^b -3x f(x) dx$$

$$\begin{aligned} \lim_{b \rightarrow \infty} \int_1^b f'(x) dx &= \lim_{b \rightarrow \infty} (f(x)) \Big|_1^b \\ &= \lim_{b \rightarrow \infty} f(b) - f(1) \\ &= 0 - 4 = -4 \end{aligned}$$

Work for problem 5(b)

$$f'(x) = -3x f(x)$$

Δx	.5	.5	.5
x	1	1.5	2
$f(x)$	4	-2	2.5
$f'(x)$	-12	9	

$$f'(1) = -3(1)(4) = -12$$

$$f'(1.5) = -3\left(\frac{3}{2}\right)(-2) = 9$$

$$f(2) \approx 2.5$$

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NO CALCULATOR ALLOWED

Work for problem 5(c)

A₂

$$\frac{dy}{dx} = -3xy$$

$$\frac{dy}{y} = -3x dx$$

$$\ln y = -\frac{3}{2}x^2 + C$$

$$y = e^{-\frac{3}{2}x^2 + C}$$

$$y = Ce^{-\frac{3}{2}x^2}$$

$$4 = Ce^{-\frac{3}{2}(1)^2}$$

$$C = \frac{4}{e^{-\frac{3}{2}}} = 4e^{\frac{3}{2}}$$

$$f(x) = y = 4e^{\frac{3}{2}}(e^{-\frac{3}{2}x^2})$$

NO CALCULATOR ALLOWED

D₁

Work for problem 5(a)

$$\begin{aligned}
 a) \int_1^{\infty} -3x f(x) dx &= \lim_{b \rightarrow \infty} \int_1^b -3x f(x) dx \\
 &= \lim_{b \rightarrow \infty} \left[(-3x)^2 - (-3x)(-3) \right]_1^b \\
 &= \lim_{b \rightarrow \infty} \left[9x^2 - 9x \right]_1^b \\
 &= \lim_{b \rightarrow \infty} 9b^2 - 9b - \cancel{9(1)} - 8 \\
 &= \boxed{\infty}
 \end{aligned}$$

$$\begin{aligned}
 f &= -3x \\
 g &= f(x) \\
 g' &= f'(x) \\
 f' &= -3
 \end{aligned}$$

Work for problem 5(b)

$$f(1) = 4$$

$$\begin{aligned}
 \Delta x &= .5 \\
 \Delta y &= \text{slope at } f_n(x_n, y_n) \cdot \Delta \\
 y_{n+\Delta x} &= y_n + \Delta y
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{1} \text{ at } (1, 4), \text{ slope} &= f'(1) = -3(1)4 \\
 &= -12
 \end{aligned}$$

$$\begin{aligned}
 \Delta y &= -12 (.5) \\
 &= -6
 \end{aligned}$$

$$\begin{aligned}
 y_{1.5} &= 4 + -6 \\
 &= -2 \quad \therefore f(1.5) = -2 \\
 &\quad (1.5, -2)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \text{ at } (1.5, -2), \text{ slope} &= f'(1.5) = -3(1.5)(-2) \\
 &= 9
 \end{aligned}$$

$$\begin{aligned}
 \Delta y &= 9 (.5) \\
 &= 4.5
 \end{aligned}$$

$$y_{2.0} = -2 + 4.5 = 2.5$$

$$\boxed{f(2) = 2.5}$$

$$\therefore (2, 2.5)$$

Work for problem 5(c)

D₂

$$\textcircled{1} \quad \frac{dy}{dx} = -3xy$$

$$\frac{1}{y} dy = -3x dx$$

$$\textcircled{2} \quad \int \frac{1}{y} dy = \int -3x dx$$

$$\textcircled{3} \quad \ln y = -\frac{3x^2}{2} + C$$

$$f(1) = 4 \quad \therefore \quad \ln 4 = -\frac{3(1)^2}{2} + C$$

$$\ln 4 = -\frac{3}{2} + C$$

$$\textcircled{4} \quad C = \ln 4 + \frac{3}{2}$$

$$\textcircled{5} \quad \ln y = -\frac{3x^2}{2} + \ln 4 + \frac{3}{2}$$

$$y = f(x) = e^{\left(-\frac{3x^2}{2} + \ln 4 + \frac{3}{2}\right)}$$