



AP[®] Calculus BC 2003 Sample Student Responses Form B

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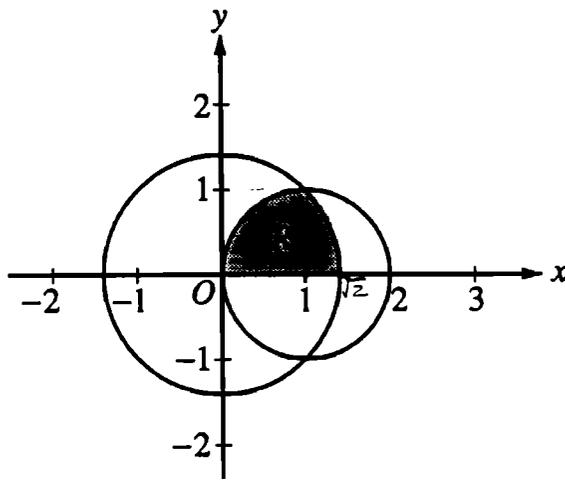
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Work for problem 2(a)

We can say that,

$$x^2 + y^2 = 2 \Rightarrow y = \sqrt{2 - x^2} \quad (y > 0)$$

$$(x-1)^2 + y^2 = 1 \Rightarrow y = \sqrt{1 - (x-1)^2} = \sqrt{2x - x^2} \quad (y > 0)$$

$$\therefore R = \int_0^1 \sqrt{2x - x^2} dx + \int_1^{\sqrt{2}} \sqrt{2 - x^2} dx$$

(Note that $y=0$ at $x=\sqrt{2}$, in the graph of circle $x^2 + y^2 = 2$.)

Work for problem 2(b)

We can say that

$$x^2 + y^2 = 2 \Rightarrow x = \sqrt{2 - y^2} \quad (x > 0)$$

$$(x-1)^2 + y^2 = 1 \Rightarrow x = 1 - \sqrt{1 - y^2} \quad (x < 1)$$

$$\therefore R = \int_0^1 \sqrt{2 - y^2} dy - \int_0^1 (1 - \sqrt{1 - y^2}) dy = \int_0^1 (\sqrt{2 - y^2} - (1 - \sqrt{1 - y^2})) dy$$

Continue problem 2 on page 7.

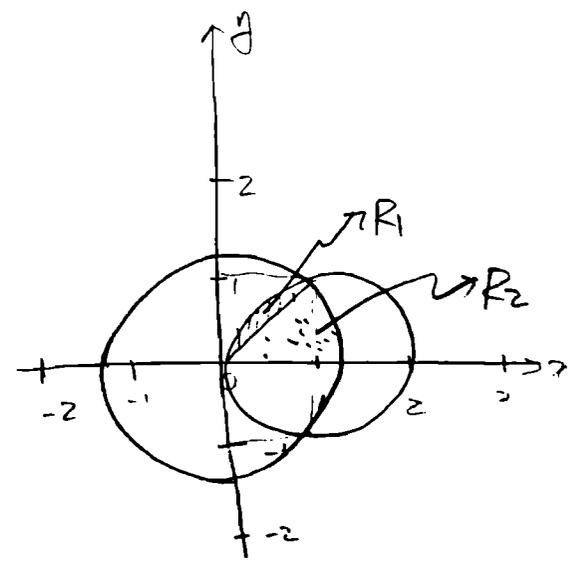
Work for problem 2(c)

Let $\begin{pmatrix} r_1 = \sqrt{2} \\ r_2 = 2\cos\theta \end{pmatrix}$

The graph of r_2 goes through $(1, 1)$ when $\theta = \frac{\pi}{4}$ ($\because \sqrt{1^2+1^2} = 2\cos\frac{\pi}{4} = \sqrt{2}$)

Also, as θ increases from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ on the graph of r_2 , r_2 draws the arc of R_2 shown the figure below.

$$\begin{aligned} \therefore S &= \int_0^{\frac{\pi}{4}} \frac{1}{2} r_1^2 d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} r_2^2 d\theta \\ &= \int_0^{\frac{\pi}{4}} d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos^2\theta d\theta \\ &= \int_0^{\frac{\pi}{4}} d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta \end{aligned}$$



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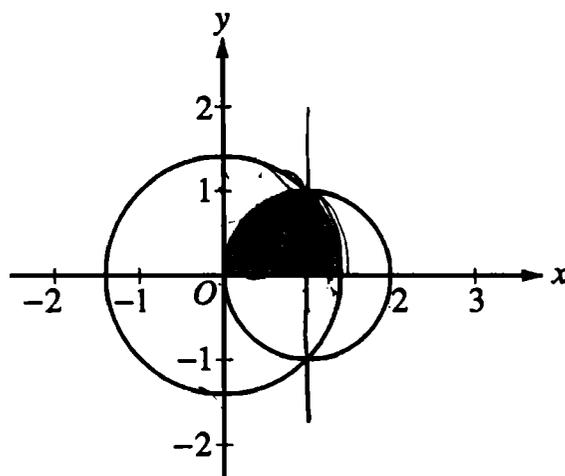
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2

2

7

$$\begin{aligned}
 & (x^2 + 2x + 1) \\
 & (x^2 + 2x) \\
 & (x^2 + 2x + 1) \\
 & (x^2 - 2x + 1) \\
 & (x^2 + 2x)
 \end{aligned}$$



Work for problem 2(a)

$$(x-1)^2 + y^2 = 1 \Rightarrow y^2 = 1 - (x-1)^2 \Rightarrow y = \sqrt{1 - (x-1)^2}$$

$$\int_0^1 \sqrt{1 - (x-1)^2} dx + \int_1^2 \sqrt{2-x} dx = 0.785 + 0.368 = 1.153$$

$$x^2 + y^2 = 2 \quad y = 2 - x$$

$$y = \sqrt{2-x}$$

Work for problem 2(b)

$$x^2 + y^2 = 2$$

$$\rightarrow x^2 = 2 - y^2$$

$$\rightarrow x = \sqrt{2 - y^2}$$

$$(x-1)^2 + y^2 = 1$$

$$\rightarrow (x-1)^2 = 1 - y^2$$

$$\rightarrow x-1 = -\sqrt{1-y^2}$$

$$x = -\sqrt{1-y^2} + 1$$

$$\int_0^1 \sqrt{2-y^2} - (-\sqrt{1-y^2} + 1) dy = 1.071$$

Continue problem 2 on page 7.

$$r = 2 \cos \theta$$

Work for problem 2(c)

$$S = \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (2 \cos \theta)^2 d\theta$$

$$R = \frac{1}{2} \int_{0.817}^{1.5} (2 \cos \theta)^2 d\theta + \frac{1}{2} \int_0^{0.817} (\sqrt{2})^2 d\theta$$

$$= 0.255 + 0.817$$

$$= 1.072$$

GO ON TO THE NEXT PAGE.