



## **AP<sup>®</sup> Calculus BC (Operational) 2004 Sample Student Responses**

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## NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = \sin\left(5x + \frac{\pi}{4}\right)$$

$$f(0) = \frac{\sqrt{2}}{2}$$

$$f'(x) = \cos\left(5x + \frac{\pi}{4}\right) \cdot 5$$

$$f'(0) = \frac{5\sqrt{2}}{2}$$

$$f''(x) = -5\sin\left(5x + \frac{\pi}{4}\right) \cdot 5$$

$$f''(0) = -\frac{25\sqrt{2}}{2}$$

$$f'''(x) = -25\cos\left(5x + \frac{\pi}{4}\right) \cdot 5$$

$$f'''(0) = -\frac{125\sqrt{2}}{2}$$

$$\begin{aligned} \therefore p(x) &= \frac{\sqrt{2}}{2} + \frac{\frac{5\sqrt{2}}{2}}{1!}x - \frac{\frac{25\sqrt{2}}{2}}{2!}x^2 - \frac{\frac{125\sqrt{2}}{2}}{3!}x^3 \\ &= \frac{\sqrt{2}}{2} + \frac{5}{2}\sqrt{2}x - \frac{25}{4}\sqrt{2}x^2 - \frac{125}{12}\sqrt{2}x^3 \end{aligned}$$

Work for problem 6(b)

$$\left| f^{(n)}(0) \right| = \frac{5^n \sqrt{2}}{2}$$

signs alternate every 2 terms

$$\therefore \frac{22+1}{4} = 5 \dots [2] \text{ - remainder}$$

\(\therefore\) sign negative

$$\therefore \text{coefficient} = -\frac{5^{22}\sqrt{2}}{22!} = -\frac{5^{22}\sqrt{2}}{2 \cdot 22!}$$

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Continue problem 6 on page 15.

Work for problem 6(c)

$$|f(\frac{1}{10}) - P(\frac{1}{10})| = \frac{f^4(c)}{4!} (\frac{1}{10})^4 \quad \text{for } 0 < c < \frac{1}{10}$$

$$f^4(c) = 125 \cdot \sin(5c + \frac{\pi}{4}) \cdot 5 \quad \text{for } 0 < c < \frac{1}{10}$$

$$\frac{\pi}{4} < 5c + \frac{\pi}{4} < \frac{1}{2} + \frac{\pi}{4} = \frac{2+\pi}{4} < \pi$$

$$\therefore \sin(5c + \frac{\pi}{4}) > 0 \quad \text{and} \quad \sin(5c + \frac{\pi}{4}) < 1$$

$$\therefore f^4(c) < 125 \cdot 5 = 625$$

$$\therefore \frac{f^4(c)}{4!} (\frac{1}{10})^4 < \frac{625}{24} \cdot \frac{1}{10000} = \frac{1}{24 \cdot 16} = \frac{1}{384} < \frac{1}{100}$$

$$\therefore |f(\frac{1}{10}) - P(\frac{1}{10})| < \frac{1}{100}$$

Work for problem 6(d)

$$G(x) = \int_0^x f(t) dt$$

$$\therefore G'(x) = f(x)$$

$$G''(x) = f'(x)$$

} fundamental theorem of calculus.

Taylor polynomial for G about x=0 is ~~the~~ antiderivative of P(x).

$$\therefore P(x) = \frac{\sqrt{2}}{2} + \frac{5}{2}\sqrt{2}x - \frac{25}{4}\sqrt{2}x^2 - \frac{125}{12}\sqrt{2}x^3$$

let R(x) be the ~~the~~ Taylor polynomial for G about x=0

$$\therefore R(x) = \frac{\sqrt{2}}{2}x + \frac{5}{4}\sqrt{2}x^2 - \frac{25}{12}\sqrt{2}x^3$$

END OF EXAMINATION

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = \sin\left(5x + \frac{\pi}{4}\right)$$

$$f(0) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$f'(x) = 5\cos\left(5x + \frac{\pi}{4}\right) \quad f'(0) = \frac{5}{\sqrt{2}}$$

$$f''(x) = -25\sin\left(5x + \frac{\pi}{4}\right) \quad f''(0) = -\frac{25}{\sqrt{2}}$$

$$f'''(x) = -125\cos\left(5x + \frac{\pi}{4}\right) \quad f'''(0) = -\frac{125}{\sqrt{2}}$$

$$P(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$$

$$P(x) = \frac{1}{\sqrt{2}} + \frac{5x}{\sqrt{2}} - \frac{25x^2}{2\sqrt{2}} - \frac{125x^3}{6\sqrt{2}}$$

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Work for problem 6(b)

$$x^n = \frac{5^n}{n! \sqrt{2}}$$

$$\text{coefficient of } x^n = \frac{5^n}{n! \sqrt{2}}$$

$$\text{coefficient of } x^{22} = \frac{5^{22}}{22! \sqrt{2}}$$

Continue problem 6 on page 15.

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NO CALCULATOR ALLOWED

Work for problem 6(c)

Lagrange error

$$|f(\frac{1}{10}) - P(\frac{1}{10})| \leq \frac{f^{(n+1)}(z) (x-c)^{n+1}}{(n+1)!}$$

$$\leq \frac{f^{(4)}(z) (\frac{1}{10})^4}{4!} < \frac{1}{100}$$

$$\leq \frac{625 (\sin(\frac{5}{10} + \frac{\pi}{4}))}{24} \frac{1}{10000} < \frac{1}{100}$$

$$\leq \frac{625}{10000} \sin(\frac{2+\pi}{4}) < \frac{1}{100}$$

$$\leq \frac{625 \sin(\frac{2+\pi}{4})}{240000} < \frac{1}{100}$$

$$|f(\frac{1}{10}) - P(\frac{1}{10})| \leq \frac{\sin(\frac{2+\pi}{4})}{384} < \frac{1}{100}$$

$f^{(4)}(z) = 625(\sin(5z + \frac{\pi}{4}))$

$\sin(\frac{2+\pi}{4}) < 1$   
 $\frac{1}{384} < \frac{1}{100}$

Work for problem 6(d)

$$G(x) = c + \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}}$$

$$G(0) = 0 \quad c = 0$$
~~$$G(x) = \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}}$$~~

$$G(x) = \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}}$$

END OF EXAMINATION

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