



## AP Calculus BC 2000 Student Samples

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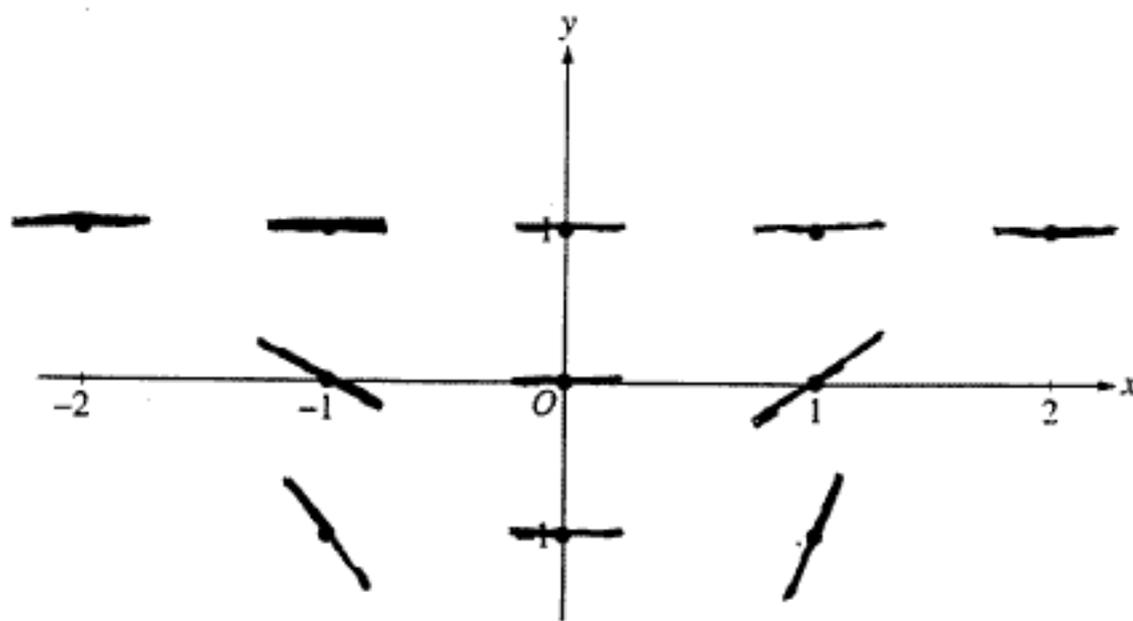
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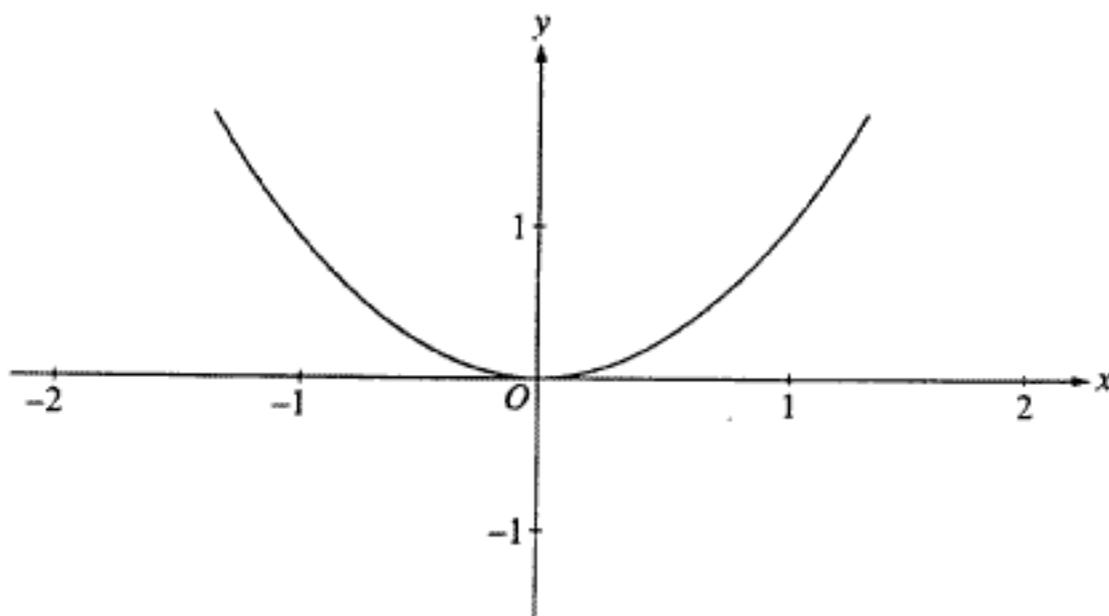
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Work for problem 6(a)

- $(0,0) = 0$
- $(0,1) = 0$
- $(0,-1) = 0$
- $(1,0) = 1$
- $(1,1) = 0$
- $(1,-1) = 4$
- $(2,1) = 0$
- $(-1,1) = 0$
- $(-2,1) = 0$
- $(-1,0) = -1$
- $(-1,-1) = -4$



Work for problem 6(b)



This graph passes through the points  $(-1, 1)$  and  $(1, 1)$  and has a slope whose absolute value is greater than 0. However, in the slope field, the slope at these points is 0. Therefore, this graph cannot match the differential equation.

Continue problem 6 on page 15.

Work for problem 6(c)

$$\frac{dy}{dx} = x(y-1)^2$$

$$\frac{dy}{(y-1)^2} = x dx$$

integrate:  $\frac{-1}{(y-1)} = \frac{1}{2}x^2 + C$

$$\frac{-1}{(-1-1)} = \frac{1}{2}(0)^2 + C$$

$$\frac{1}{2} = 0 + C$$

$$C = \frac{1}{2}$$

$$\frac{-1}{y-1} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$\frac{-1}{\frac{1}{2}(x^2+1)} = y-1$$

$$y = \frac{-2}{x^2+1} + 1$$

Work for problem 6(d)

when  $x = 0$ ,  $y = -1$

as  $x$  approaches  $\infty$  or  $-\infty$ ,  $y$  goes to 1

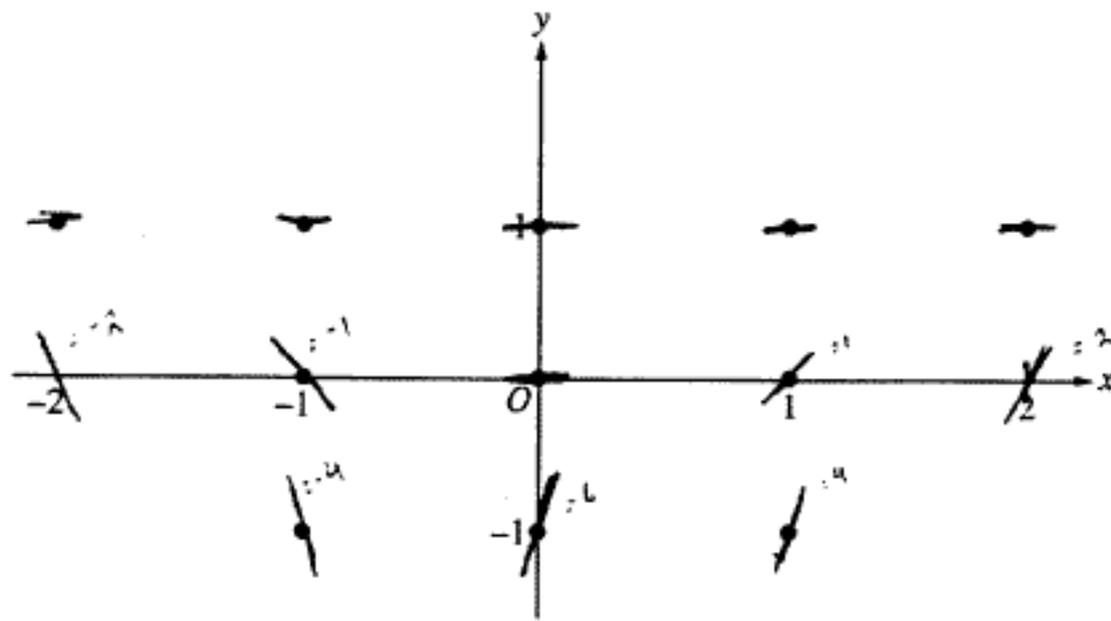
$$[-1, 1)$$

END OF EXAMINATION

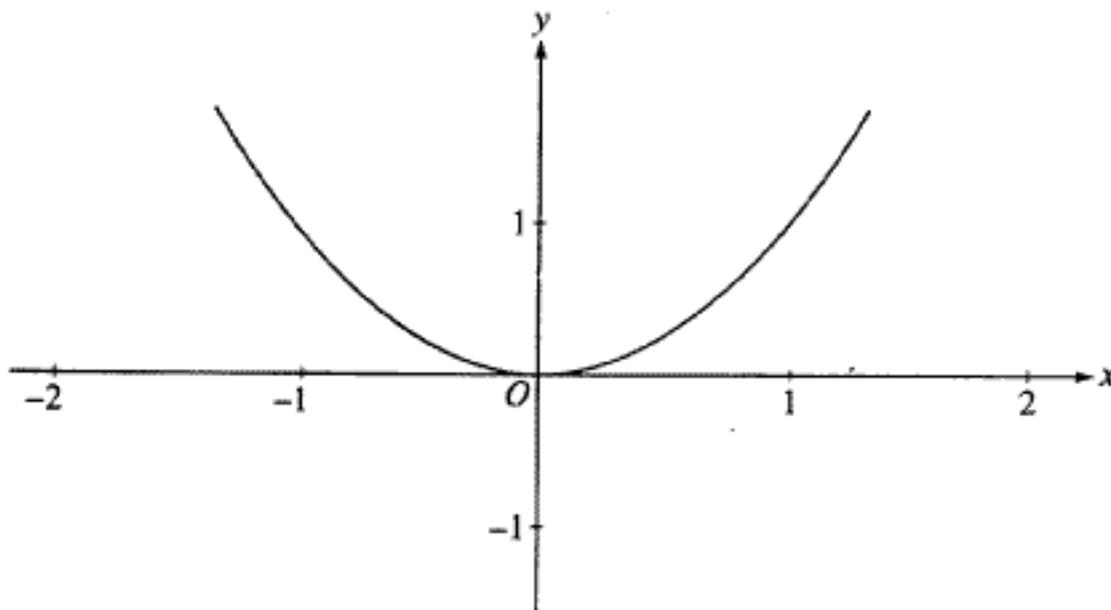
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Work for problem 6(a)



Work for problem 6(b)



This graph can not be a solution because at the points  $(-1, 1)$  and  $(1, 1)$  the slope of the function must be zero. In this particular graph there is a slope not equaling 0 at these two points.

Continue problem 6 on page 15.

Work for problem 6(c)

$$-\frac{dy}{(y-1)^2} = x dx$$

$$u = y-1$$

$$\int \frac{du}{u^2} = \int x dx$$

$$= -u^{-1} = \frac{1}{2}x^2$$

$$\frac{-1}{(y-1)} = \frac{1}{2}x^2 + C$$

$$-\frac{1}{(-1-1)} = \frac{1}{2}(0)^2 + C$$

$$-\frac{1}{-2} = C \quad C = \frac{1}{2}$$

$$-\frac{1}{(y-1)} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{1}{2}(x^2+1)$$

$$-(y-1) = \frac{2}{x^2+1}$$

$$y = -\frac{2}{(x^2+1)} + 1$$

Work for problem 6(d)

$$\text{range} = -1 < y < 1$$

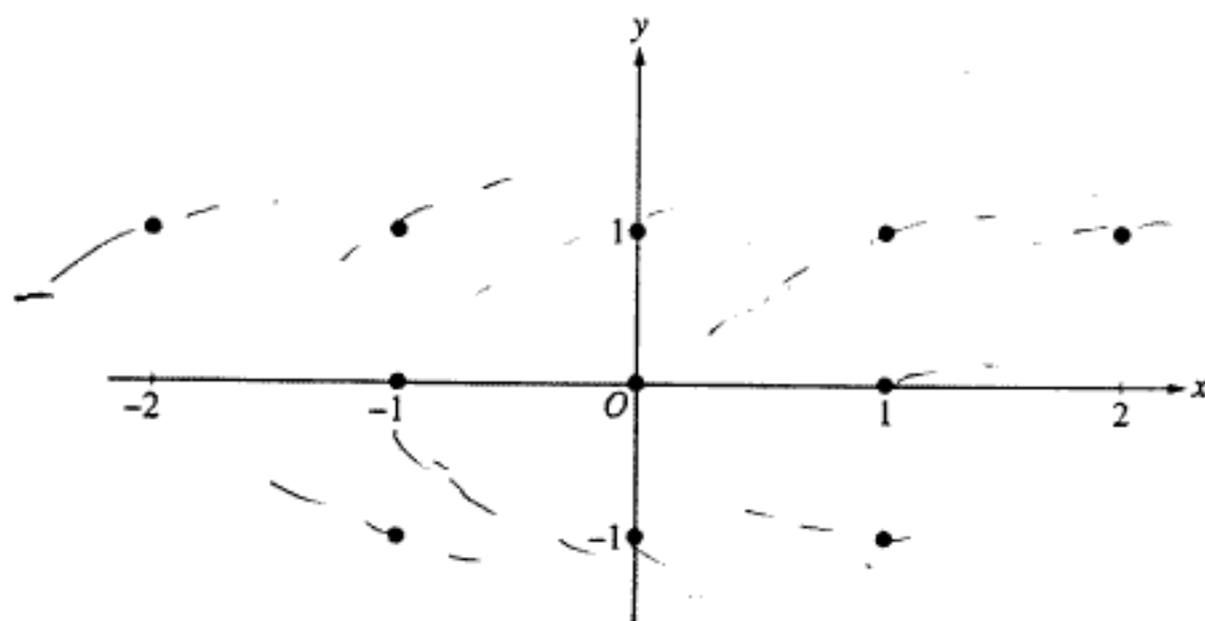
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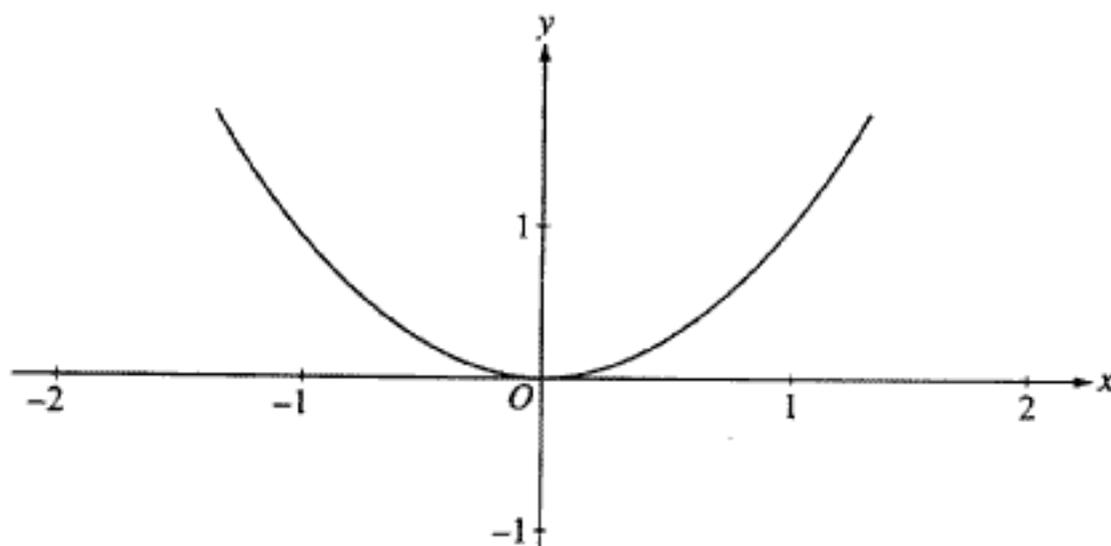
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Work for problem 6(a)

$$\frac{dy}{dx} = x(y-1)^2 \rightarrow y^2 - x - 2xy + x$$



Work for problem 6(b)



Parabolas such as the one shown above have 2 values of  $x$  for each unique value  $y$ . However, the differential equation from above shows that there are more values for  $y$  than  $x$ , so the shape ~~so be~~ should ~~be~~ lie sideways.

Continue problem 6 on page 15.

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F<sub>2</sub>

Work for problem 6(c)

$f(0) = -1$

$$\int \frac{dy}{(y-1)^2} = \int x dx \rightarrow \int \frac{dy}{y^2 - 2y + 1} = \int x dx$$

$$\frac{(y-1)(y+1)}{x}$$

$$\frac{1}{2}x^2 = -(y-1)^{-1} + C$$

$$\frac{1}{2}x^2 + C = -\frac{1}{y-1} \rightarrow (0, -1): 0 + C = -\frac{1}{-1-1} \rightarrow C = -\frac{1}{(-2)} = \frac{1}{2}$$

$$\frac{1}{2}x^2 + \frac{1}{2} = -\frac{1}{y-1} \rightarrow -\frac{1}{2}x^2 - \frac{1}{2} = \frac{1}{y-1} \rightarrow \frac{-x^2 - 1}{2} = \frac{1}{y-1}$$

$$\frac{2}{-(x^2+1)} = (y-1) \rightarrow \frac{-2}{x^2+1} + 1 = y \Rightarrow \frac{-2+x^2+1}{x^2+1} \Rightarrow \frac{x^2-1}{x^2+1} = y$$

Work for problem 6(d)

range:

\* every value of  $y$  allowed because  $x^2+1$  can never equal 0, thus  $y$  will never be  $\infty$

END OF EXAMINATION

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