



## AP Calculus BC 2000 Student Samples

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## CALCULUS BC

## SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$\vec{v} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \right\rangle$$

$$\vec{a} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle = \left\langle 2t^{-3}, -2t^{-3} \right\rangle$$

$$\Rightarrow \vec{a}(3) = \left\langle \frac{2}{27}, -\frac{2}{27} \right\rangle$$

Work for problem 4(b)

$$x(t) = \int \left( \frac{dx}{dt} \right) dt = \int (1 - t^{-2}) dt = t + t^{-1} + C$$

$$x(1) = 2 + C = 2 \Rightarrow C = 0$$

$$y(t) = \int \left( \frac{dy}{dt} \right) dt = \int (2 + t^{-2}) dt = 2t - t^{-1} + C$$

$$y(1) = 1 + C = 6 \Rightarrow C = 5$$

$$\Rightarrow (x(t), y(t)) = (t + t^{-1}, 2t - t^{-1} + 5)$$

$$\therefore (x(3), y(3)) = \left( \frac{10}{3}, \frac{32}{3} \right)$$

Continue problem 4 on page 11.

Work for problem 4(c)

$$\frac{\frac{dy}{dx}}{\frac{dy}{dt}} = \frac{2 + t^{-2}}{1 - t^{-2}} = \frac{2t^2 + 1}{t^2 - 1} = 8$$

$$\Leftrightarrow 6t^2 = 9$$

$$\Leftrightarrow t = \sqrt{\frac{3}{2}}, \quad t > 0$$

Work for problem 4(d)

$$\lim_{t \rightarrow \infty} \left( \frac{dy}{dx} \right) = \lim_{t \rightarrow \infty} \left( \frac{2t^2 + 1}{t^2 - 1} \right) \quad ; \quad \begin{array}{l} \text{both numerator \& denominator} \\ \text{are undefined, so by} \\ \text{L'Hopital's Rule,} \end{array}$$

$$= \lim_{t \rightarrow \infty} \left( \frac{4t}{2t} \right) = \lim_{t \rightarrow \infty} 2 = 2$$

$\therefore$  The slope of the line that  $y(x)$  approaches is 2

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

C<sub>1</sub>

No calculator is allowed for these problems.

Work for problem 4(a)

$$\text{velocity} = (1 - t^{-2}, 2 + t^{-2})$$

$$\text{acc} = (2t^{-3}, -2t^{-3})$$

$$\left(\frac{2}{t^3}, -\frac{2}{t^3}\right) \text{ at } t=3$$

$$\boxed{\left(\frac{2}{27}, -\frac{2}{27}\right)}$$

Work for problem 4(b)

$$\text{velocity} = (1 - t^{-2}, 2 + t^{-2})$$

$$x(t) = \int (1 - t^{-2}) dt$$

$$y(t) = \int (2 + t^{-2}) dt$$

$$t=1 \text{ pos } (2, 6)$$

$$t + t^{-1} + C$$

$$2t - t^{-1} + C$$

$$2 = 1 + 1 + C$$

$$6 = 2 - 1 + C$$

$$C=0$$

$$6 = 1 + C$$

$$x(t) = t + \frac{1}{t}$$

$$y(t) = 2t + \frac{1}{t} + 5$$

$$6 + \frac{1}{3} + 5 =$$

at  $t=3$

position

$$\boxed{\left(\frac{4}{3}, 11\frac{1}{3}\right)}$$

Continue problem 4 on page 11.

4 4 4 4 4 4 4 4 4 4

C2

Work for problem 4(c)

$$\frac{dx}{dt} = 1 - t^{-2} \quad \frac{dy}{dt} = 2 + t^{-2}$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{2 + t^{-2}}{1 - t^{-2}} \quad \frac{2 + t^{-2}}{1 - t^{-2}} = 8$$

$$2 + t^{-2} = 8 - 8t^{-2}$$

$$9t^{-2} = 6$$

$$\frac{9}{t^2} = 6$$

$$6t^2 = 9$$

$$t^2 = \frac{9}{6}$$

$$t = \sqrt{\frac{9}{6}} = \boxed{\frac{3}{\sqrt{6}}}$$

Work for problem 4(d)

$$\lim_{t \rightarrow \infty} \frac{2 + t^{-2}}{1 - t^{-2}} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

take deriv  
of top + bottom  
L'Hopital's rule

$$\lim_{t \rightarrow \infty} \frac{-2t^{-3}}{2t^{-3}} = \boxed{-1}$$

CALCULUS BC  
SECTION II, Part B  
Time—45 minutes  
Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$v(t) = \left( 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \right)$$

$$a(t) = \left( \frac{1}{2t^3}, -\frac{1}{2t^3} \right)$$

$$\Rightarrow v(t) = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$a(t) = \begin{pmatrix} dx \\ dy \end{pmatrix}$$

$$a(3) = \left( \frac{1}{2(3)^3}, -\frac{1}{2(3)^3} \right)$$

$$a(3) = \left( \frac{1}{54}, -\frac{1}{54} \right)$$

Work for problem 4(b)

$$v(t) = \left( 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \right) \Rightarrow v(t) = \begin{pmatrix} dx \\ dy \end{pmatrix}$$

$$p(t) = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$p(t) = \left( t + \frac{1}{t}, 2t - \frac{1}{t} \right)$$

$$p(3) = \left( 3\frac{1}{3}, 5\frac{2}{3} \right)$$

Continue problem 4 on page 11.

Work for problem 4(c)

$$\frac{dy}{dx} = 8 = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

$$8 = \frac{8}{t^2} = 2 + \frac{1}{t^2}$$

$$6 = \frac{9}{t^2}$$

$$6t^2 = 9$$

$$t^2 = \frac{3}{2}$$

$$t = \frac{\sqrt{3}}{\sqrt{2}}$$

Work for problem 4(d)

$$\lim_{t \rightarrow \infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$