



AP[®] Calculus BC

2003 Sample Student Responses

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NO CALCULATOR ALLOWED

Work for problem 6(c)

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$f'(x) = \sum_{n=0}^{\infty} \frac{-2n(-1)^n x^{2n-1}}{(2n+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$xy' + y = x \sum_{n=0}^{\infty} \frac{-2n(-1)^n x^{2n-1}}{(2n+1)!} + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{-2n(-1)^n x^{2n}}{(2n+1)!} + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{-2n(-1)^n x^{2n} + (-1)^n x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \cos x$$

Thus $y = f(x)$ is a solution to the differential equation

$$xy' + y = \cos x$$

END OF EXAMINATION

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Work for problem 6(a)

$$f(x) = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$$

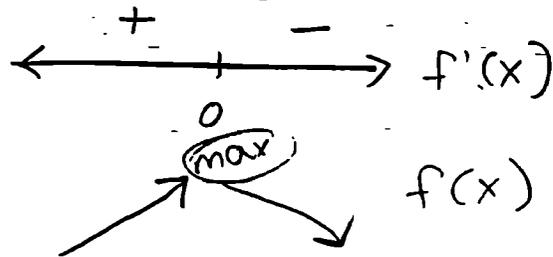
$$f'(x) = -\frac{2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \dots$$

$$f'(0) = 0$$

$$f''(x) = -\frac{2}{3!} + \frac{12x^2}{5!} - \frac{30x^4}{7!} + \dots$$

$$f''(0) = -\frac{2}{3!} = -\frac{1}{3}$$

$$f''(0) = -\frac{1}{3}$$



Local max @ x=0

Work for problem 6(b)

Series is alternating so
 error < first neglected term

first neglected term = $\frac{x^4}{5!}$

@ 1 = $\frac{1}{5!} = \frac{1}{120}$

$\frac{1}{120} < \frac{1}{100}$

$\frac{x^5}{120}$

Continue problem 6 on page 15.

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NO CALCULATOR ALLOWED

Work for problem 6(c) -

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$xy' + y = \cos x$$

$$xy' = \cos x - y$$

$$y' = \frac{\cos x - y}{x}$$

$$\frac{dy}{dx} = \frac{\cos x - y}{x}$$

$$\frac{\cos x}{x} + \frac{dy}{dx} = -\frac{y}{x}$$

$$\int \frac{\cos x}{x} + \frac{dy}{y} = \int -\frac{1}{x} dx$$

$$-\frac{\cos x}{x} + \int \frac{dy}{y} = \int -\frac{dx}{x}$$

$$-\ln|y| + c_1 = -\ln|x| + c_2$$

$$x \cdot \left(-\frac{2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \dots \right) + \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} \right)$$

$$-\frac{2x^2}{3!} + \frac{4x^4}{5!} - \frac{6x^6}{7!} + \dots + 1 - \frac{x^2}{3!} + \frac{x^4}{5!}$$

$$1 - \left(\frac{3x^2}{3!} \right) + \left(\frac{5x^4}{5!} \right) - \dots$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \cos x$$

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