



Student Performance Q&A: 2001 AP[®] Calculus Free-Response Questions

The following comments are provided by the Chief Faculty Consultant regarding the 2001 free-response questions for AP Calculus AB and BC. *They are intended to assist AP workshop consultants as they develop training sessions to help teachers better prepare their students for the AP Exams.* They give an overview of each question and its performance, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also included. Consultants are encouraged to use their expertise to create strategies for teachers to improve student performance in specific areas.

AB Question 1

This problem presented the student with two regions in the first quadrant.

Parts (a) and (b) of this problem required the student to use definite integrals to find the areas of two regions. Part (c) asked the student to find the volume of a solid with known cross sections (in this case, a solid of revolution). A calculator equation-solver (numerical or graphical) was needed to find the intersection of two curves that form part of the boundary of the regions in question. It was expected that students would also use the numerical integration capabilities of their calculators to evaluate the definite integrals.

One aspect of this problem was the variety of avenues to the solutions for the areas of regions R and S . The fact that the two regions were complementary enhanced the student's choices. Although this made the grading of the question somewhat more complex, it perhaps also made the problem more accessible to a wider range of students.

In each definite integral, one limit came from the computation of the point of intersection of the two curves. Students were expected to write the limit to three decimal places when showing the setup of the definite integral. In all problems involving decimal approximations, however, the final answer should be correct to three decimal places, which often requires that intermediate calculations be stored in the calculator to a greater precision. Students need to be aware of the danger of premature rounding of intermediate results when doing such computations.

Students scored fairly well on this problem with almost 20 percent receiving a 9. A score of 6 was also common. A sizeable number of students tried unsuccessfully to find the area of region R by integration with respect to x using only one region, earning no points on part (a), but then finishing the rest of the problem correctly. Other students were able to find both areas but could not handle the integral for the volume.

The mean score was 4.84 indicating a reasonably strong overall performance.

AB/BC Question 2

This problem presented the student with tabular data representing water temperature readings in a pond recorded at regular time intervals.

Part (a) asked for an approximation of a derivative (with appropriate units) and part (b) asked for a (trapezoidal) approximation of a definite integral representing the average value of the temperature using this tabular data. In part (c), a function model for the water temperature was introduced. The student was asked to calculate a derivative analytically and to provide an interpretation of its meaning in this physical context. Part (d) required the student to calculate the average value of the function model over the time interval in part (b). This problem reflected the increased emphasis on working with multiple representations of functions. It illustrated two ways of working with data: approximations based on actual data values and analytic work based on a model approximating the data.

To receive full credit for this problem, the student needed to distinguish between the discrete model and the continuous model. Many students successfully computed the derivative and average value in parts (c) and (d) but were clearly uncomfortable with how to handle the approximations using the tabular values. In particular, in part (a), after computing an appropriate difference quotient, some students went on to do unnecessary and often incorrect additional computation with the difference quotient. An important part of explaining the meaning of $P'(12)$ was the interpretation of the sign of the numerical answer. Students needed to clearly say that the temperature was decreasing when $t = 12$, not just restate the meaning of the derivative as a “rate of change.”

The mean score was 3.48 for the AB students and 5.27 for the BC students, indicating a weak performance by the AB population but a strong performance by the BC group. Over 23 percent of the AB students earned no points on this problem, while less than 8 percent of the BC students did so.

AB/BC Question 3

This problem presented the student with a car’s initial velocity and a piecewise linear graphical representation of the car’s acceleration over a time interval. The different parts of the problem asked for interpretations and conclusions regarding the car’s velocity. This required the student to both recognize the acceleration graph as that of the derivative

of the velocity and to reason using this graphical representation of the derivative. It is possible that a student might have chosen to obtain a piecewise formula for the acceleration function in order to work analytically, but it would have been more efficient to reason directly from the graph.

Part (a) asked for an interpretation of the rate of change of the velocity at a given time (obtained by the sign of the value of the acceleration function). Parts (b), (c), and (d) each required calculating accumulated changes in velocity in terms of definite integrals of the acceleration function, all of which could be computed directly by summing the signed areas of triangles, rectangles and/or trapezoids. Part (c) also involved an analysis of local extrema and endpoint analysis in finding the time at which an absolute maximum velocity occurs. All parts of the problem asked for supporting reasons and justifications for the students' conclusions.

More points were given for justifications and explanations (5 points out of 9) than on any previous AP Calculus problem. As one result, this problem took a long time to grade since many students wrote long paragraphs on each of the four parts yet sometimes still earned no points. A common misconception was that many students thought that the given curve was the velocity function instead of the acceleration function. Another common mistake was to forget to include the initial condition when computing velocities.

The mean score was a very disappointing 2.40 for the AB students, the lowest mean among the six AB problems. Only 2 percent of the approximately 144,000 AB students earned all 9 points. Moreover, almost 32 percent of the AB population earned no points on this problem, a disappointment considering the emphasis placed on previous AP Calculus Exams on analyzing the behavior of a function given the graph of its derivative. The mean score for the BC students was 4.17.

AB/BC Question 4

This problem presented the student with a derivative of a function h and an initial value of the function. Given this information, a student might have attempted to determine the function h explicitly by first antidifferentiating h' and then using the given initial condition to determine the constant of integration. However, the function h had a discontinuity at $x = 0$ and so the initial condition only determined the function for $x > 0$. For all parts of this problem the student could have used information supplied by the given derivative h' or its derivative h'' .

Part (a) asked for the critical values and for an extrema analysis with justification. This justification could have involved either the first or second derivative tests. Part (b) asked for a concavity analysis that could be in terms of the second derivative or in terms of the increasing/decreasing behavior of the first derivative. Part (c) required the student to find the equation of the line tangent to the graph of h at the point defined by the initial condition. Part (d) tied parts (b) and (c) together by asking for a geometric analysis of the

relationship of the line tangent to the graph of h found in part (c) by using the concavity information from part (b).

The question generated a full range of scores, although there was a decided shift in the mean to the lower end of the range, especially among the AB students. Students who failed to support their answers with complete justifications or explanations lost several points.

One of the crucial issues in this problem was to recognize and handle the discontinuity in the derivative at $x = 0$. This was particularly true for those students attempting an extrema analysis with a sign chart. The use of the second derivative test avoided the subtlety at $x = 0$. Some students drew incorrect conclusions from their analysis of a sign chart even when they recognized the discontinuity at $x = 0$, seeming to treat all functions as polynomials with alternating maxima and minima. The most common error in part (b) was failing to exclude $x = 0$ from the intervals on which the graph of h was concave up. In part (d) it was important for the student to reason globally about the concavity of the graph of h for all $x > 4$, not give just a local statement about the concavity at the point where $x = 4$.

The mean score was 3.05 for the AB students and 4.70 for the BC students. Because of the increased emphasis on justifications and explanations in this problem, scores of 9 were rare. Only 1 percent of the AB students and 4 percent of the BC students received full credit. On the other hand, most students were able to earn some points.

AB Question 5

This problem presented the student with a cubic polynomial function involving three undetermined coefficients, along with the location of a local minimum and a point of inflection of the graph.

Part (a) asked the student to determine the values of two of the coefficients using the given information. This problem was a slight variation of the routine exercise of finding the extrema and points of inflection of a given function. Part (b) provided additional information — the value of a definite integral of the cubic function — and the student was asked to use this to find the remaining coefficient of the polynomial.

Typical student errors on this problem were either algebraic, incorrect antidifferentiation of the constant term k , or incorrect use of the Fundamental Theorem of Calculus to find the value of the parameter k . Apparently students need more practice in dealing algebraically with signed numbers and in working with parameters as opposed to variables.

The mean score was 6.11, indicating a good performance by the AB students. Over 54 percent of the students received a score of 8 or 9. As a result, however, the problem did not discriminate well except at the 1/2 cut score.

AB Question 6

This problem presented the student with a point on the graph of a function and an expression for the derivative of the function in terms of both x and y .

Part (a) asked the student to find the second derivative at the given point. Since the first derivative was given in terms of x and y , the second derivative could have been found using implicit differentiation. The information given in this problem also defined a differential equation. Part (b) asked the student to find an explicit formula for the function by solving this separable differential equation with initial condition. It is possible that a student might have first answered part (b) and then used this explicit solution to solve part (a) without using implicit differentiation.

Many students confused the problem in part (a) with the similar problem of finding the derivative of an implicitly defined function. Chain rule errors lost both differentiation points and in almost every case made students ineligible for the answer point.

The standard for part (b) represented what has become the usual approach to grading separable differential equation problems. The first 3 points were for mechanical operations of separating the variables and antidifferentiating the two sides. The last 3 points were for finding the particular solution satisfying the given initial condition. Inclusion of the constant of integration was critical because without it the student could not proceed from a general solution to the particular solution.

The mean score was 3.64, indicating a relatively good performance, and much better than the separable differential equation problem in 2000. Despite the fact that this was the fourth separable differential equation problem in the past five years, almost 30 percent of the AB population received a score of 0 or left the problem blank. However, almost 9 percent of the students received full credit.

BC Question 1

This problem involved parametric motion in the plane. It presented the student with the velocity components of an object moving along a curve in the plane and the object's position at a given time.

Part (a) asked for an equation of the line tangent to the curve, which required the student to determine the slope from the velocity components. Part (b) asked for a calculation of the object's speed and part (c) asked for the total distance traveled by the object over a given time interval. Part (d) required the student to integrate each of the velocity components and use the position of the object at $t = 2$ to calculate the coordinates of the object's position at a specific time. The velocity components of the object were functions that required the use of the numerical integration capabilities of a calculator in parts (c) and (d).

Parts (a) and (b) both involved a series of numerical calculations to obtain the final answer. This was again a problem in which premature rounding of intermediate results could adversely affect the accuracy of the final answer. Typical errors in these parts included finding dx/dy rather than dy/dx , not substituting appropriate values of t , and mistakenly using the expressions given in the stem as the functions for x and y rather than for the derivatives. In part (c), some students tried to use the formula for arclength of a graph rather than setting up a distance integral for the motion of a particle.

Part (d) involved similar work using the Fundamental Theorem of Calculus to find the x -coordinate and the y -coordinate of the object at time $t = 3$. Many students earned few points in this part because they worked with an inappropriate interval, tried to solve the integrals analytically, or failed to include the initial position with the integral.

The mean score on this question was a very disappointing 2.69, reflecting a poor performance by the BC students. Over half the students received no more than 2 points.

BC Question 5

This problem presented the student with a differential equation relating a function f to its derivative f' . The student was also presented with both an initial condition and a limiting condition (in other words, the asymptotic behavior of $f(x)$ as x approaches infinity).

Part (a) asked for an improper integral of the expression for f' given by the differential equation. The solution of this problem required the student to recognize the integrand as being f' and use the Fundamental Theorem of Calculus to employ both the initial and limiting conditions in the calculation. Part (b) asked the student to use Euler's method to approximate the value of the function at another point. Part (c) asked the student to explicitly solve the given separable differential equation with initial condition. It is possible that a student might have used this explicit solution to approach part (a) and/or part (b).

There has not been an improper integral on the free response section in many years, which may have contributed to the difficulty some students had with part (a). The form of the integrand made some students attempt to use integration by parts. Euler's method is still relatively new to the course description and, as such, is new to many teachers and textbooks. Though most students attempted to do part (b), it was apparent that they were not as well prepared for it as for solving the separable differential equation in part (c). The standard for grading the solution of the separable differential equation was comparable to that used for AB6 except only 5 points were available instead of 6. Again, the use of a constant of integration was fundamental in proceeding from a general solution to the particular solution.

This question was split graded with parts (a) and (b) being BC material only (improper integrals and Euler's method) and part (c) contributing to the Calculus AB subscore grade

(solution for a separable differential equation). The mean score on parts (a) and (b) was 1.91 (out of 4), with about 23 percent earning all 4 points. The mean score on the last part was 3.70 (out of 5) for a total mean of 5.61. This indicates a reasonably strong student performance. This was a fairly easy separable differential equation and almost 50 percent of the students earned all 5 points in part (c).

BC Question 6

This problem presented the student with a power series representation for a function.

Part (a) asked for the interval of convergence of the given power series. Part (b) asked for a limit of an indeterminate form $\left(\frac{0}{0}\right)$ involving the function defined by the power series.

A student might have used L'Hôpital's Rule or simplified the indeterminate form. In either case, some formal manipulation of the power series was needed, either by differentiation or by algebraic means. Part (c) also required the student to formally manipulate the given power series in a way that could have been used to calculate the value of a definite integral. This resulted in a geometric series that the student was asked to evaluate in part (d).

Most students got a good start on part (a) with a ratio (root) test setup, algebra work, and a computation of the limit. Students were explicitly asked to show the work that led to their conclusions. A common error was to leave out any notation of limit in proceeding from the ratio (root) test setup to the value for the radius of convergence. Many students never checked the endpoints for convergence or divergence, or never even indicated in their work that they needed to consider these points to answer part (a).

Many students got part (b) using one of the three basic approaches: algebraic manipulation of the series, L'Hôpital's Rule, or interpreting this limit as $f'(0)$ and then differentiating the series term-by-term. The most common error in part (c) was failing to obtain the terms for the definite integral after doing the correct computation of an antiderivative of the infinite series.

All BC examinations in recent years have had a problem on series, and student performances have generally been weak. The mean score on this question was 3.68, again relatively weak, but slightly better than the result on the 2000 BC exam.