



**AP<sup>®</sup> Calculus BC  
2004 Sample Student Responses  
Form B**

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A,

Work for problem 2(a)

$$T(x) = f(2) + f'(2)(x-2) + \frac{f''(2)(x-2)^2}{2} + \frac{f'''(2)(x-2)^3}{3!}$$

$$= 7 + 0 - 9(x-2)^2 - 3(x-2)^3$$

$$f(2) = 7$$

$$\frac{f''(2)(x-2)^2}{2} = -9(x-2)^2$$

$$f''(2) = -18$$

Work for problem 2(b)

Yes there is. to have this Taylor polynomial,  $f'(2) = 0$  must be true

$f''(2) = -18$ , which is negative, meaning curve is concave down



if curve is concave down, and 1st derivative is 0, it is a relative max

$$f(2) \text{ is a relative maximum}$$

Continue problem 2 on page 7.

Work for problem 2(c)

$$T(0) = 7 - 9(0-2)^2 - 3(0-2)^3 = -5$$

$$f(0) \approx T(0) = -5$$

No, there isn't, because we do not know any of the derivatives at 0,  
so nothing can be determined.

Work for problem 2(d)

$$\frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1} = \text{max error}$$

$$\frac{f^{(4)}(c)}{(4)!} (x-2)^4 \leq \frac{6}{4!} (x-2)^4 = \frac{(x-2)^4}{4} = \text{max error}$$

$$\frac{(0-2)^4}{4} = 4 = \text{max error}$$

$-5 \pm 4 < 0$ , all values within error range is still negative,  
so  $f(0)$  is negative

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C

Work for problem 2(a)

$$T(x) = f(2) + f'(2)(x-2) + \frac{f''(2)(x-2)^2}{2!} + \frac{f'''(2)(x-2)^3}{3!}$$

$$\therefore T(x) = 7 + 0 - 9(x-2)^2 - 3(x-2)^3$$

$$f(2) = 7$$

$$\frac{f''(2)(x-2)^2}{2!} = -9(x-2)^2$$

$$f''(2) = -9(2)$$

$$f''(2) = -18$$

Work for problem 2(b)

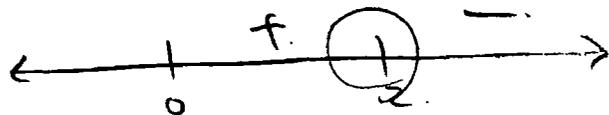
$$T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$$

$$T'(x) = -18(x-2) - 9(x-2)^2 = 0$$

critical pts.  $\rightarrow x = 2, 0$

$$T'(1) = 9$$

$$T'(3) = -27$$



relative max.

yes there is enough information to determine whether  $f$  has a critical pt. at  $x=2$ .

$f(2)$  is a relative maximum. by the 1st derivative test.

Continue problem 2 on page 7.

Work for problem 2(c)

$$f(0) \approx T(0) = 7 - 9(0-2)^2 - 3(0-2)^3$$

$$f(0) \approx T(0) = -5$$

so you can't determine whether  $f$  has a critical point at  $x=0$  because we do not know  $f$ .  $T(x)$  is a Taylor polynomial about  $x=2$ , thus it is only an approximation.  $x=0$  is too far from where  $T(x)$  is centered, thus the approximation is very inaccurate.

Work for problem 2(d)

$$\text{Lagrange error bound} = \frac{f^{(n+1)}(z) (x-c)^{(n+1)}}{(n+1)!}$$

$$= \frac{f^{(4)}(z) (x-2)^4}{4!}$$

when  $x=0$ .

$$= \frac{f^{(4)}(z) (-2)^4}{4!} = \frac{6 (-2)^4}{4!} = -4$$

The max error bound is  $-4$ , thus  $f(0)$  lies somewhere is  $T(0) \pm \text{error}$ , somewhere in  $-9$  to  $-1$ . Thus  $f(0)$  is negative.

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