



AP[®] Calculus BC 2001 Sample Student Responses

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NO CALCULATOR ALLOWED

B₁

Work for problem 6(a)

Ratio Test

$$L = \lim_{n \rightarrow \infty} \left| \frac{(n+2)x^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{(n+1)x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x}{3} \cdot \frac{n+2}{n+1} \right| = \left| \frac{x}{3} \right| < 1$$

$$-1 < \frac{x}{3} < 1 \quad \therefore -3 < x < 3$$

$$x = -3: \sum_{n=0}^{\infty} \frac{(n+1)(-3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(n+1)(-1)^n (3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)}{3} \quad \text{Diverges}$$

$$x = 3: \sum_{n=0}^{\infty} \frac{(n+1)3^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n+1}{3} \quad \text{Diverges}$$

$$\boxed{(-3, 3)}$$

Work for problem 6(b)

$$\frac{f(x) - \frac{1}{3}}{x} = \frac{2}{3^2} + \frac{3}{3^3}x + \dots + \frac{(n+2)}{3^{n+2}}x^n$$

$$\lim_{x \rightarrow 0} \frac{f(x) - \frac{1}{3}}{x} = \frac{2}{3^2} = \boxed{\frac{2}{9}}$$

Work for problem 6(c)

$$\int_0^1 f(x) dx = \left[\frac{1}{3}x + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \dots + \frac{x^n}{3^n} \right] \Big|_0^1$$

$$= \boxed{\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^n}}$$

Work for problem 6(d)

The series determined in part c is a geometric series with initial term $t_1 = \frac{1}{3}$ and a ratio $r = \frac{1}{3}$.

$$\text{Sum} = \frac{t_1}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$$

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NO CALCULATOR ALLOWED

D₁

Work for problem 6(a)

$$\lim_{x \rightarrow 0} \frac{\frac{(n+1+1)x^{n+1}}{3^{n+1+1}}}{\frac{(n+1)x^n}{3^{n+1}}} \quad \lim_{x \rightarrow 0} \frac{(n+2)(x^n \cdot x^1)}{3^n \cdot 3^2} \cdot \frac{3^1 \cdot 3^1}{(n+1)(x^n)} \Rightarrow \lim_{n \rightarrow \infty} \frac{n+2}{n+1} \cdot \frac{x}{3}$$

$$|\frac{x}{3}| < 1 \quad -1 < \frac{x}{3} < 1 \quad -3 < x < 3$$

Check endpoints

$$\sum \frac{n+1}{3^{n+1}} (-3)^n \quad \text{converges} \quad \sum \frac{n+1}{3^{n+1}} (3^n) \quad \text{diverges}$$

$$\boxed{-3 \leq x < 3}$$

Work for problem 6(b)

$$\lim_{x \rightarrow 0} \frac{f(x) - \frac{1}{3}}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\frac{2}{3^2}x + \frac{3}{3^3}x^2 + \dots + \frac{n+1}{3^{n+1}}x^n + \dots}{x}$$

$$\lim_{x \rightarrow 0} \frac{2}{3^2} + \frac{3}{3^3}x + \dots + \frac{n+1}{3^{n+1}}x^{n-1}$$

$$= \frac{2}{3^2} = \boxed{\frac{2}{9}}$$

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NO CALCULATOR ALLOWED

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Work for problem 6(c)

$$g(x) = \frac{1}{3}x + \frac{2x^2}{3^2 \cdot 2} + \frac{2x^3}{3^3 \cdot 2}$$

$$g(x) = \frac{1}{3}x \Big|_0^1 + \frac{x^2}{9} \Big|_0^1 + \frac{x^3}{27} \Big|_0^1$$

$$g(x) = \frac{1}{3} + \frac{1}{9} + \frac{1}{27}$$

Work for problem 6(d)

$$S_n = \frac{a_1}{1-r}$$

$$a_1 = \frac{1}{3} \quad r = \frac{1}{3}$$

$$S_n = \frac{\frac{1}{3}}{1 - \frac{1}{3}}$$

$$S_n = \frac{\frac{1}{3}}{\frac{3}{3} - \frac{1}{3}}$$

$$S_n = \frac{\frac{1}{3}}{\frac{2}{3}}$$

$$S_n = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2} = \boxed{\frac{1}{2}}$$