



AP[®] Calculus BC 2002 Scoring Guidelines Form B

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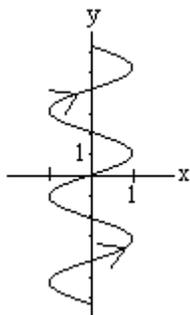
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Question 1

A particle moves in the xy -plane so that its position at any time t , for $-\pi \leq t \leq \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.

- (a) Sketch the path of the particle in the xy -plane provided. Indicate the direction of motion along the path.
- (b) Find the range of $x(t)$ and the range of $y(t)$.
- (c) Find the smallest positive value of t for which the x -coordinate of the particle is a local maximum. What is the speed of the particle at this time?
- (d) Is the distance traveled by the particle from $t = -\pi$ to $t = \pi$ greater than 5π ? Justify your answer.

(a)



- 1 : graph
 three cycles of sine
 2 { x between -1 and 1
 y between -2π and 2π
 1 : direction

(b) $-1 \leq x(t) \leq 1$
 $-2\pi \leq y(t) \leq 2\pi$

- 2 { 1 : closed interval for $x(t)$
 1 : closed interval for $y(t)$

(c) $x'(t) = 3 \cos 3t = 0$
 $3t = \frac{\pi}{2}; t = \frac{\pi}{6}$
 Speed = $\sqrt{9 \cos^2(3t) + 4}$
 At $t = \frac{\pi}{6}$,
 Speed = $\sqrt{9 \cos^2\left(\frac{\pi}{2}\right) + 4} = 2$

- 3 { 1 : $x'(t) = 3 \cos 3t = 0$
 1 : solves for t
 1 : speed at student's time

(d) Distance = $\int_{-\pi}^{\pi} \sqrt{9 \cos^2(3t) + 4} dt$
 $= 17.973 > 5\pi$

- 2 { 1 : integral for distance
 1 : conclusion with justification

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Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

- (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?
- (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
- (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
- (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$
 so the amount is not increasing at this time.

1 : answer with reason

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$
 $t = (5 \ln 3)^2 = 30.174$
 $P'(t)$ is negative for $0 < t < (5 \ln 3)^2$ and positive for $t > (5 \ln 3)^2$. Therefore there is a minimum at $t = (5 \ln 3)^2$.

3 { 1 : sets $P'(t) = 0$
 1 : solves for t
 1 : justification

(c) $P(30.174) = 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}}) dt$
 $= 35.104 < 40$, so the lake is safe.

3 { 1 : integrand
 1 : limits
 1 : conclusion with reason
 based on integral of $P'(t)$

(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.

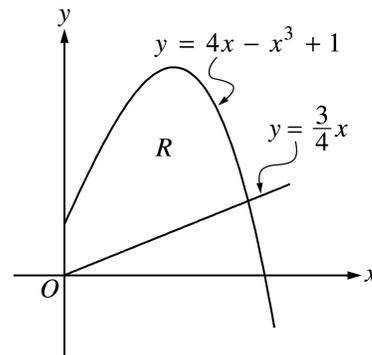
2 { 1 : slope of tangent line
 1 : answer

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Question 3

Let R be the region in the first quadrant bounded by the y -axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the x -axis.
- (c) Write an expression involving one or more integrals that gives the perimeter of R . Do not evaluate.



Region R

$$4x - x^3 + 1 = \frac{3}{4}x \text{ when } x = 1.94045 = A$$

$$\begin{aligned} \text{(a) Area} &= \int_0^A \left(4x - x^3 + 1 - \frac{3}{4}x \right) dx \\ &= 4.514 \text{ or } 4.515 \end{aligned}$$

$$3 \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) Volume

$$\begin{aligned} &= \pi \int_0^A \left((4x - x^3 + 1)^2 - \left(\frac{3}{4}x \right)^2 \right) dx \\ &= 18.291\pi \text{ or } 57.463 \end{aligned}$$

$$3 \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

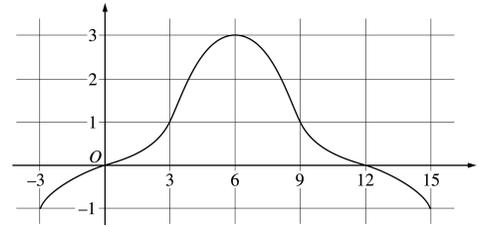
$$\begin{aligned} \text{(c) Perimeter} &= 1 + \sqrt{(1.940)^2 + (1.455)^2} \\ &\quad + \int_0^A \sqrt{1 + (4 - 3x^2)^2} dx \end{aligned}$$

$$3 \begin{cases} 1 : \text{uses } y' = 4 - 3x^2 \text{ in integrand} \\ 1 : \text{arc length integral} \\ 1 : \text{answer} \end{cases}$$

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Question 4

The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let



Graph of f

$$g(x) = 5 + \int_6^x f(t) dt \text{ for } -3 \leq x \leq 15.$$

- (a) Find $g(6)$, $g'(6)$, and $g''(6)$.
 (b) On what intervals is g decreasing? Justify your answer.
 (c) On what intervals is the graph of g concave down? Justify your answer.
 (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t) dt$ using six subintervals of length $\Delta t = 3$.

(a) $g(6) = 5 + \int_6^6 f(t) dt = 5$
 $g'(6) = f(6) = 3$
 $g''(6) = f'(6) = 0$

$$3 \left\{ \begin{array}{l} 1 : g(6) \\ 1 : g'(6) \\ 1 : g''(6) \end{array} \right.$$

(b) g is decreasing on $[-3, 0]$ and $[12, 15]$ since
 $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$.

$$3 \left\{ \begin{array}{l} 1 : [-3, 0] \\ 1 : [12, 15] \\ 1 : \text{justification} \end{array} \right.$$

(c) The graph of g is concave down on $(6, 15)$ since
 $g' = f$ is decreasing on this interval.

$$2 \left\{ \begin{array}{l} 1 : \text{interval} \\ 1 : \text{justification} \end{array} \right.$$

(d) $\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1)$
 $= 12$

1 : trapezoidal method

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Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

- (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f . Find the x -coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.
- (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

(a) $\frac{dy}{dx} = 0$ when $x = 3$

$$\left. \frac{d^2y}{dx^2} \right|_{(3,-2)} = \left. \frac{-y - y'(3-x)}{y^2} \right|_{(3,-2)} = \frac{1}{2},$$

so f has a local minimum at this point.

or

Because f is continuous for $1 < x < 5$, there is an interval containing $x = 3$ on which

$y < 0$. On this interval, $\frac{dy}{dx}$ is negative to the left of $x = 3$ and $\frac{dy}{dx}$ is positive to the

right of $x = 3$. Therefore f has a local minimum at $x = 3$.

(b) $y \, dy = (3-x) \, dx$

$$\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C$$

$$8 = 18 - 18 + C; C = 8$$

$$y^2 = 6x - x^2 + 16$$

$$y = -\sqrt{6x - x^2 + 16}$$

$$3 \left\{ \begin{array}{l} 1 : x = 3 \\ 1 : \text{local minimum} \\ 1 : \text{justification} \end{array} \right.$$

$$6 \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } g(6) = -4 \\ 1 : \text{solves for } y \end{array} \right.$$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

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Question 6

The Maclaurin series for $\ln\left(\frac{1}{1-x}\right)$ is $\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \leq x < 1$.

(a) Find the Maclaurin series for $\ln\left(\frac{1}{1+3x}\right)$ and determine the interval of convergence.

(b) Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

(c) Give a value of p such that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverges. Give reasons why your value of p is correct.

(d) Give a value of p such that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges. Give reasons why your value of p is correct.

(a)
$$\ln\left(\frac{1}{1+3x}\right) = \ln\left(\frac{1}{1-(-3x)}\right)$$

$$= \sum_{n=1}^{\infty} \frac{(-3x)^n}{n} \text{ or } \sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n} x^n$$

We must have $-1 \leq -3x < 1$, so interval of convergence is $-\frac{1}{3} < x \leq \frac{1}{3}$.

2 $\left\{ \begin{array}{l} 1 : \text{series} \\ 1 : \text{interval of convergence} \end{array} \right.$

(b)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \ln\left(\frac{1}{1-(-1)}\right) = \ln\left(\frac{1}{2}\right)$$

1 : answer

(c) Some p such that $0 < p \leq \frac{1}{2}$ because

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$$
 converges by AST, but the p -series
$$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$$
 diverges for $2p \leq 1$.

3 $\left\{ \begin{array}{l} 1 : \text{correct } p \\ 1 : \text{reason why } \sum \frac{(-1)^n}{n^p} \text{ converges} \\ 1 : \text{reason why } \sum \frac{1}{n^{2p}} \text{ diverges} \end{array} \right.$

(d) Some p such that $\frac{1}{2} < p \leq 1$ because the

p -series
$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$
 diverges for $p \leq 1$ and the p -series
$$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$$
 converges for $2p > 1$.

3 $\left\{ \begin{array}{l} 1 : \text{correct } p \\ 1 : \text{reason why } \sum \frac{1}{n^p} \text{ diverges} \\ 1 : \text{reason why } \sum \frac{1}{n^{2p}} \text{ converges} \end{array} \right.$