



AP[®] Calculus BC 2006 Free-Response Questions

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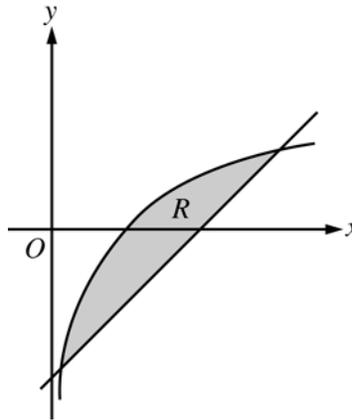
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2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

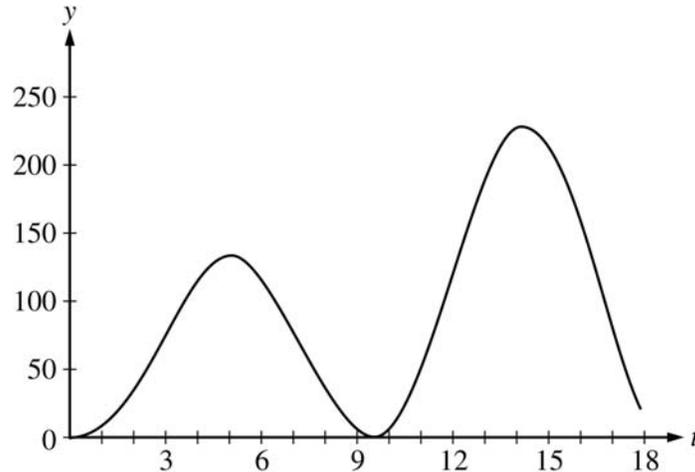
A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.
 - (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.
 - (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y -axis.
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2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \leq t \leq 18$ hours. The graph of $y = L(t)$ is shown above.
- (a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \leq t \leq 18$ hours.
- (b) Traffic engineers will consider turn restrictions when $L(t) \geq 150$ cars per hour. Find all values of t for which $L(t) \geq 150$ and compute the average value of L over this time interval. Indicate units of measure.
- (c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.
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3. An object moving along a curve in the xy -plane is at position $(x(t), y(t))$ at time t , where

$$\frac{dx}{dt} = \sin^{-1}(1 - 2e^{-t}) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

- Find the acceleration vector and the speed of the object at time $t = 2$.
 - The curve has a vertical tangent line at one point. At what time t is the object at this point?
 - Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t \rightarrow \infty} m(t)$.
 - The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c .
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END OF PART A OF SECTION II

2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS**CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3**

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.
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5. Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.
- (a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.
- (b) Is it possible for the x -axis to be tangent to the graph of f at some point? Explain why or why not.
- (c) Find the second-degree Taylor polynomial for f about $x = -1$.
- (d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.
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6. The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1} + \cdots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers x for which the series converges.

- (a) Find the interval of convergence of the power series for f . Justify your answer.
- (b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.
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END OF EXAM